

INDEX

- absolute value functions 355–60, 392–7
- absolute value inequalities 81–2, 359–60
- absolute values 48, 53–4
 - equations involving 56–7, 358–9
 - properties 53
- acceleration 412, 455, 456, 463–4
- acute angles 592
- addition rule
 - for mutually exclusive events 495
 - of probability 499–501
- adjacent side (triangle) 226
- algebraic expressions
 - expanding 16
 - factorising 19, 20
 - simplifying 14, 27
- algebraic fractions, operations with 28
- alternate angles 224
- ambiguous case of the sine rule 224, 253–4
- amplitude
 - periodic functions 592, 629
 - trigonometric functions 631–2
- angle of depression 224, 241
- angle of elevation 224, 240
- angle of inclination of a line 140, 164–6
- angles
 - cosine rule for 257–8
 - finding in a right-angled triangle 236
 - measured in degrees, minutes and seconds 228
 - and sides of a triangle 250
- angles of any magnitude 592–7
 - 1st quadrant: acute angles (between 0° and 90°) 592
 - 2nd quadrant: obtuse angles (between 90° and 180°) 592–3
 - 3rd quadrant: angles between 180° and 270° 593
 - 4th quadrant: angles between 270° and 360° 593
- ASTC rule 594
 - negative angle 596–7
- arc, length of an 274–6
- area of a sector 277–8
- area of a triangle, sine formula for 260–1
- arrangements 88
- ASTC rule 594–6
 - negative angles 597, 619
 - radians 619–20
- asymptotes 348, 352–4, 561
- average rate of change 412, 425, 426, 430, 453, 454
- axis of symmetry 140
 - of a parabola 181–3
- bases 4, 549
 - and number systems 555
- bearings 224, 242
 - compass 242
 - true 243–4
- binomial expansion 88, 122
- binomial expressions 4, 17
- binomial products 4, 17, 21
 - and Pascal's triangle 120–3
 - special products 18
- Body Mass Index 68
- break-even points 140, 171, 211–12
- Briggs, Henry 553
- calculus 412
 - and motion 438, 462
 - notation 430
 - origins 462
- Cardano, Girolamo 499
- Cartesian form of a function 398
- Cartesian plane 146
- centre
 - periodic functions 592, 629
 - trigonometric functions 632
- chain rule 412, 446–7
- circles 361
 - with centre $(0, 0)$ 361
 - with centre (a, b) 362–3
 - parametric equations of 402–4
- coefficients
 - Pascal's triangle 123–6
 - polynomials 140, 204, 302–8
- co-interior angles are supplementary 224
- combinations 88, 112–15
 - and Pascal's triangle 119–20
- combined functions 372–4
- common logarithms 551–2
- compass bearings 224, 242
- complement 486, 496
- complementary angle results 601

- complementary angles 601
- complementary events 496
- completing the square, to solve quadratic equations 62–3
- composite function rule 446
- composite functions 348, 374–5
- concavity
 - cubic functions, and point of inflection 197–8
 - parabola, and turning point 179
- conditional probability 486, 513–17
 - and independent events 517–18
 - multi-stage events 505–6
 - product rule 516–17
- constant of variation 348
- constant term (polynomial) 140, 204
- continuous functions 348, 352
- continuous random variables 660–1
- converse of the factor theorem 293
- corresponding angles 224
- $\cos^{-1}(-x)$ 646–7
- cosecant function 627
- cosecant ratio 599
- cosine function 625–6
 - inverse 643
- cosine ratio 227
 - exact ratios 272–3
 - quadrants 248–9
- cosine rule 256–8
 - applications 263–5
 - for angles 257–8
 - for right-angled triangles 257
- cotangent function 628
- cotangent ratio 599
- counting techniques 88–9
 - factorial notation 98–100
 - fundamental counting principle 89–91
 - pigeonhole principle 94–7
- cubic equations
 - finding 201–2
 - roots and coefficients 304–7
- cubic functions 140, 196–202
 - concavity 197–8
 - point of inflection 197–9
- curve, gradient of a 416–20
- decay constant 570
- deceleration 464
- decimal degrees, and degrees-minutes-seconds 228–30
- decreasing graphs 154, 197
- decreasing line 413
- degree (polynomial) 140, 204
- degree-minute-second (DMS) key (calculators) 228–30
- degrees (angles) 228
 - converting to radians 270–1
 - decimal 228–30
 - exact trigonometric ratios 272, 273
 - and radians 269–70
- dependent variable 140, 141
- derivative
 - of a constant multiple of a function 435
 - of e^{ax} 547
 - of e^x 545
 - of $[f(x)]^n$ 446
 - of k 434
 - of ke^x 545–6
 - of kx 434
 - of kx^n 435
 - of a sum of functions 435
 - of x^n 434–6
- derivative functions 417–18, 421, 429
 - exponential functions 541
 - sketching 418–19
- derivatives, and indices 439
- Descartes, Rene 146
- difference quotient 429–32
- difference of two squares 18
 - factorising 25
- differentiability 412, 421–3
 - at a point 422
- differentiation 412, 421
 - chain rule 412, 446–7
 - exponential functions 545–7
 - product rule 412, 448–51
 - quotient rule 412, 451–2
 - short methods of 434–6
- differentiation from first principles 412, 424, 429, 431–2
 - estimating the gradient of a tangent 426–7, 429
 - gradient of a secant 424–6, 427–8
- Diophantus of Alexandria 13, 50
- direct proportion 140
- direct variation 140, 159
- Dirichlet principle 97
- discontinuous functions 348, 352–4, 421
- discrete probability distributions 662–4
 - properties 664–7
- discrete probability function 662
- discrete random variables 660–1
- discriminant 140, 187, 188–9
 - and the parabola 189–90
 - and roots of a quadratic equation 188–90
- displacement 412, 455, 456, 463
- distributive law 16, 19
- dividend 288, 289
- divisor 288, 289

- domain 140, 142, 153
 - hyperbolas 352–4
 - and increasing and decreasing graphs 154
 - inverse functions 326
 - linear functions 161–2
 - restricting, inverse functions 288, 327–9
- double angle identities 609–10
- double roots of the equation 317
- elimination method, simultaneous
 - equations 70–1
- equally likely outcomes 486, 487
- equation of a circle
 - with centre $(0, 0)$ 361
 - with centre (a, b) 362–3
- equations 48–9
 - cubic 201–2
 - exponential 48, 57–8, 567–8
 - involving absolute values 56–7, 358–9
 - linear 169–70
 - linear simultaneous 69–70
 - polynomial 205–6
 - quadratic 48, 61, 62–3, 64–5
 - substitution into formulas to solve 66
 - trigonometric 635–8
- equations of a semicircle with centre $(0, 0)$ 364–5
- equilateral triangle 225
- Euler, Leonhard 150, 541
- Euler's number, e 536, 540–3
- even functions 140, 155
- event 486
- exact trigonometric ratios 272
 - in radians 272
- expanding expressions 16
- expected value 660, 673–8
- exponential decay 536, 570–4
 - and the environment 578
 - modified 579–81
- exponential equations 48, 57–8, 567–8
- exponential functions 536–40
 - derivative functions 541
 - differentiation 545–7
 - and Euler's number, e 541–3
 - reflections 539
 - sketching 536–9
 - $y = a^x$ 537–8
 - $y = e^x$ 542
- exponential growth 536, 570–4
 - modified 579–81
- expressions 4
- exterior angle in any triangle 225
- factor theorem 288, 293–4
 - converse 293
- factorial notation 88, 98–100, 103
- factorisation 4, 19
 - by grouping in pairs 20
 - mixed 26
 - to solve quadratic equations 61
- factorising
 - difference of two squares 25
 - perfect squares 24
 - polynomials 296–7
 - trinomials 21–2, 23–4
- factors 4
- Fermat, Pierre de 499
- Fibonacci sequence 126
- 1st quadrant: acute angles (between 0° and 90°)
 - 248, 592
- formulas
 - substitution into 30
 - substitution into to solve equations 66
- 4th quadrant – angles between 270° and 360° 593
- fractional indices 9–12
- 'function of a function' rule 446
- function notation 148–50
- functions 140, 142–3
 - absolute value 355–60, 392–7
 - combined 372–4
 - composite 374–5
 - cubic 140, 196–202
 - domain and range 153, 154
 - even and odd 140, 155–6
 - exponential 536–40
 - gradient (derivative) 417–19
 - horizontal line test 144–6, 323–4
 - hyperbolas 349, 351–2
 - increasing and decreasing graphs 154
 - intercepts 152
 - inverse 326–9
 - inverse relation of 320–4
 - linear 159–62
 - logarithmic 536, 561–4
 - many-to-one 142
 - one-to-one 140, 142, 144–6, 323, 326
 - piecewise 140, 149, 355–6
 - polynomial 204–5
 - products of 379–80
 - properties of 152–6
 - quadratic 140, 177–80, 181–3
 - reciprocal 381–7
 - reflection of 366–71
 - substituting pronumerals into 150
 - sum of 377–8
 - trigonometric 624–33
 - vertical line test 143–4
- fundamental counting principle 88, 89–91

- Galileo 438, 553, 640
- Gauss, Carl Friedrich 50
- general cubic function 199–201
- general trigonometric functions 632–3
- generalised pigeonhole principle 95–7
- geometry results 224–6
- gradient 140
 - of a curve 416–20
 - of parallel lines 173–4
 - of perpendicular lines 175, 442
 - of a secant 412, 424–6, 427–8
 - of a straight line 163–6, 413–15
 - of a tangent 412, 416, 426–7, 429
- gradient formula 164
- gradient functions 417–18
 - sketching 418–19
- gradient–intercept equation of a straight line 167
- graphing
 - cubic functions 196–201
 - exponential functions 540
 - inverse of a function 322–4
 - linear functions 160–1
 - logarithmic functions 564
 - polynomial functions 206–7
 - quadratic functions 177–80
 - see also* sketching
- graphs
 - intersection of 209–13
 - $y = -f(-x)$ 369–70
 - $y = -f(x)$ 356–7, 366–7
 - $y = f(-x)$ 367–9
 - $y = f(x)$ 366–70
 - $y = kx^3$ 197
- growth constant 570
- Hermite 541
- horizontal line test 140, 144–6, 288, 323–4
- horizontal lines 161–2
- horizontal point of inflection 472
- Huygens, Christiaan 438, 499
- Hypatia 50
- hyperbolas 348, 349, 351–2
 - asymptotes 352–4
 - as discontinuous functions 352–4
 - as reciprocal functions 381
- hypotenuse 226
- identity 592, 602
- increasing graphs 154, 197
- increasing line 413
- independent events 486, 504
 - and conditional probability 517–18
 - product rule 504–5
- independent variable 140, 141
- index laws 4–5
- indices 4
 - and derivatives 439
 - fractional 9–12
 - zero and negative 7–8
- inequalities 48
 - involving absolute values 81–2, 359–60
 - involving the unknown in the denominator 76–80
 - on a number line 51–2
 - quadratic 48, 74–5, 185–6
 - solving 51–2
- instantaneous rate of change 412, 426, 429, 453, 455
- intercepts of the graph of a function 140, 152
- intersection of graphs
 - break-even points 211–12
 - intersecting lines 210
 - intersecting lines and parabolas 212–13
 - solving equations graphically 209–10
- intersection (set notation) 488
- interval notation 140, 153
- inverse cosine function 643
- inverse functions 288, 563, 564
 - domain and range 326
 - notation 326
 - restricting the domain 327–9
- inverse of a function 320–1
 - graphing 322–4
 - horizontal line test 323–4
- inverse of an inverse trigonometric functions 649–50
- inverse proportion 348
- inverse sine function 642, 644
- inverse tangent function 643
- inverse trigonometric functions 592, 642–4
 - inverse of 649–50
 - properties 645–9
- inverse variation 348–51
- isosceles triangle 225
- Kepler, Johannes 553
- leading coefficient (polynomial) 140, 204
- leading term (polynomial) 140, 204
- Leaning Tower of Pisa 239
- Leibniz, Gottfried 412, 430
- length of an arc 274–6
- limiting behaviour of polynomials 312–14
- limits 412, 428–9
- linear equations, finding 169–70
- linear functions 140, 159–62
 - applications 171
 - domain and range 161
 - gradient and y-intercept 167
 - graphing 160–1
 - horizontal and vertical lines 161–2
 - parametric form 399–401
 - point of intersection 210

- linear simultaneous equations 69–70, 210
- locally straight curves 426–7
- logarithm laws 556–8
- logarithmic functions 536, 561–4
- logarithmic scales 565
- logarithms 536, 549–51
 - change of base 558–9
 - common 551–2
 - natural 551–2
 - origins 553
 - properties 551
- long division 288
 - polynomials 288, 289–91
- longest side (triangle) 250
- loss 212

- Malthus, Thomas 571, 579
- many-to-many relation 141, 143
- many-to-one function 142
- many-to-one relation 141
- mathematical verbs xvii
- maximum turning point 179, 420, 472
- mean 673–8
- minimum turning point 179, 420, 472
- minutes (angles) 228
- modified exponential growth and decay 579–81
- Mohammed Un-Musa Al-Khwarezmi 13
- monic polynomials 140, 204, 302, 304, 307
- monotonic decreasing 288, 324, 327
- monotonic increasing 288, 324, 327
- motion
 - and calculus 438, 462
 - in a straight line 462–8
- motion graphs 464–8
- multi-stage experiments 502–6
 - conditional probability 505–6
 - independent events 504–5
- multiple roots of the equation 316–18
- multiple roots of polynomial equations 318, 472–4
- multiplicity of roots 288, 317, 318
 - of $P(x)$ and $P'(x)$ 473–4
- mutually exclusive events 486, 494
 - addition rule for 495

- Napier, John 553
- natural (Naperian) logarithms 551–2
- negative angles (ASTC rule) 169, 597
- negative gradient 163, 413, 416
- negative indices 7–8
- Newton, Sir Isaac 412, 430, 438
- Newton's Law of Cooling 579–81
- non-linear simultaneous equations 70–1
- non-mutually exclusive events 496, 499–501
- normal 412, 442
 - normals to a curve 442–3
 - not differentiable functions 421–3
 - number plane, quadrants of 248–9
- obtuse angles 592
 - trigonometric ratios 248, 249
- odd functions 140, 155–6, 199
- one-to-many relation 141
- one-to-one function 140, 142, 144–6, 323, 326
- one-to-one relation 141, 142
- opposite side (triangle) 226, 250
- ordered pairs 141
- ordered selections 88, 103, 112
- Oresme, Nicole 12
- outcome 486

- parabolas 140, 177, 178
 - axis of symmetry 181–3
 - concavity 179
 - discriminant 189–90
 - and intersecting lines 212–13
 - and quadratic inequalities 185–6
 - turning points 179–80, 182–3
- parallel lines, gradient 173–4
- parallelogram 225
- parameter 348, 398
- parametric equations of a circle 402–4
- parametric equations of a function 398
 - linear functions 399–401
 - quadratic functions 401–2
- parametric form of a function 398
- Pascal, Blaise 499
- Pascal's triangle
 - and binomial products 120–3
 - and combinations 119–20
 - properties of coefficients 123–6
- perfect squares 18
 - factorising 24
- period
 - periodic functions 592, 629
 - trigonometric functions 631–2
- periodic functions 592, 629
- permutations 88, 103–5, 112
 - involving repeated objects 107–8
 - with restrictions 106–7
- perpendicular lines, gradients 175, 442
- phase shifts, trigonometric functions 592, 631–2
- piecewise functions 140, 149, 355–6
- pigeonhole principle 94–7
- point–gradient equation of a straight line 169–70
- point of inflection (cubic functions) 140, 197–9, 472
- polynomial equations 205–6, 300–1
 - multiple roots of 472–4
 - roots and coefficients 302–8

- polynomial expressions 204–5, 289
- polynomial functions 204–5
 - graphing 206–7, 310–14
 - stationary points 472–3
- polynomials 140
 - dividing 288–91
 - factor theorem 293–4
 - factorising 296–7
 - graphs of
 - even degree 312–13, 314
 - odd degree 312–13, 314
 - limiting behaviour 312–14
 - properties 294–5
 - remainder theorem 292–3
 - zero 294
- population 660
- population mean 673
- population standard deviation 681
- positive angles (ASTC rule) 594, 619
- positive gradient 163, 413, 416
- power 4, 549
- power functions 156
- principal solution 635
- probabilities, range of 495
- probability
 - addition rule 499–501
 - conditional 486, 505–6, 513–17
 - origins of 499
 - product rule 502–6
 - theoretical 494–6
- probability distributions 660
 - discrete 662–4
 - uniform 667–9
- probability formula 487–8
- probability of an event 487
- probability trees 486, 508–10, 516
- product of roots
 - cubic equations 304–8
 - quadratic equations 302–4
 - quartic equations 307–8
- product rule
 - conditional probability 516–17
 - differentiation 412, 448–51
 - independent events 504–5
 - probability 502–6
- products of functions
 - algebraic method 379
 - multiplying graphs 380
- products to sums and differences 611–13
- profit 212
- pronominals 4
- proportionality constant 159
- Ptolemy 224
- Pythagorean identities 403, 404, 602–3
- quadrants 248–9
 - 1st – acute angles (between 0° and 90°) 248, 592
 - 2nd – obtuse angles (between 90° and 180°) 249, 592–3
 - 3rd – angles between 180° and 270° 593
 - 4th – angles between 270° and 360° 593
 - see also* ASTC rule
- quadratic equations 48, 61
 - discriminant 187, 188–90
 - finding 192–3
 - roots and coefficients 302–4
 - solving by completing the square 62–4
 - solving by factorisation 61
 - solving by quadratic formula 64–5
- quadratic formula 64–5
 - discriminant 187–90
- quadratic functions 140, 177–80
 - and axis of symmetry 181–3
 - concavity 179–80
 - parametric form 401–2
 - turning points 179–80, 182–3
- quadratic inequalities 48, 74–5
 - and the parabola 185–6
- quartic equations, roots and coefficients 307–8
- quotient 288, 289
- quotient rule 412, 451–2
- radians 224, 269
 - ASTC rule 619–20
 - converting to degrees 270–1
 - and degrees 269–70
 - exact trigonometric ratios 272–3
 - special angles 271
 - to find area of a sector 277–8
 - to find length of an arc 274–6
 - trigonometric equations 637–8
 - trigonometric functions 628–9
- random variables 660–1
- range 140, 142, 153
 - absolute value functions 356–7
 - hyperbolas 352–4
 - inverse functions 326
 - linear functions 161–2
- rate of change of y 413, 416
- rates of change 453–5
 - displacement, velocity and acceleration 455–6
 - rates involving two variables 458–60
- rationalising the binomial denominator (surds) 37–9
- rationalising the denominator (surds) 36–7
- reciprocal 348
- reciprocal functions 381–7
 - properties 383

- reciprocal trigonometric ratios 592, 599–600
- rectangle 225
- reflections of exponential functions 539
- reflections of functions 366–71
 - graph of $y = -f(-x)$ 369–70
 - graph of $y = -f(x)$ 366–7
 - graph of $y = f(-x)$ 367–9
- related rates of change 458–60
- relations 141
 - types of 141
- relative frequency 486, 491, 663
- remainder 288, 289
- remainder theorem 288, 292–3
- restricted domain 288, 327–9
- rhombus 225
- right-angled triangles
 - cosine rule 257
 - finding an angle 236
 - finding a side 232
 - sides of 226
- root of a number 4
- roots of a cubic equation 304
 - sum and product 304–7
- roots of a quadratic equation 187–90, 302
 - sum and product 302–4
- roots of a quartic equation 307–8
 - sum and product 307–8
- roots of the equation 140, 187, 300
 - multiple roots 316–17
- sample mean 673
- sample space 486–7
- sample standard deviation 681
- secant 412, 425
- secant function 627
- secant ratio 599
- 2nd quadrant: obtuse angles (between 90° and 180°)
 - 249, 592–3
- seconds (angles) 228
- sector, area of 277–8
- semicircles 363–4
- set 486
- set notation 488
- shortest side (triangle) 250
- sides of a right-angled triangle 226
 - find a side 232
- sides of a triangle 250
- simple harmonic motion 640
- simplifying
 - algebraic expressions 14, 27
 - surds 32
- simultaneous equations 48
 - linear 69–70, 210
 - non-linear 70–1
 - solving using elimination method 69–70
 - solving using substitution method 69, 70–1
 - with three unknown variables 72–3
- $\sin^{-1}(-x)$ 646
- $\sin^{-1}x + \cos^{-1}x$ 647
- sine formula for the area of a triangle 260–1
- sine function 624–5
 - inverse 642, 644
- sine ratio 227
 - exact ratios 272–3
 - quadrants 248–9
- sine rule 224, 250–2
 - ambiguous case 224, 253–4
 - applications 263–5
- sketch 140
- sketching
 - exponential functions 536–9
 - gradient (derivative) functions 417–19
 - inverse trigonometric functions 641–4
 - logarithmic functions 561–4
 - see also* graphing
- special rate of change 570
- square (geometry) 225
- square root relations 387–91
- standard deviation 660, 681–2
 - formula 682–4
- stationary points
 - curves 412, 420, 472
 - parabola 179
 - on polynomial graphs 472–3
- straight line
 - angle of inclination 164–6
 - gradient 163–4, 165–6, 413
 - gradient–intercept equation 167
 - point–gradient equation 169–70
- substitution
 - into formulas 30
 - into formulas to solve equations 66
- substitution method
 - non-linear simultaneous equations 70–1
 - simultaneous equations 69
- sum of functions 377–8
 - adding graphs 378
 - algebraic method 377
- sum of roots
 - cubic equation 204–8
 - quadratic equations 302–4
 - quartic equation 307–8
- sum of the interior angles in any triangle 225

- sums and differences of angles 605–8
- surds 4
 - operations with 33–4
 - properties 32
 - rationalising the binomial denominator 37–9
 - rationalising the denominator 36–7
 - simplifying 32
- t -formulas 592, 613–15
- $\tan^{-1}(-x)$ 647
- tangent 412, 416
- tangent function 626
 - inverse 644
- tangent identity 602
- tangent ratio 227
 - exact ratios 272–3
 - quadrants 248–9
- tangents to a curve 416, 441–2
- terms 4
- theoretical probability 494–6
- 3rd quadrant: angles between 180° and 270° 593
- trapezium 226
- tree diagrams 486, 503
- triangle
 - area of 260–1
 - cosine rule 256–8
 - naming the sides and angles 250
 - and sine rule 250–4
 - see also* right-angled triangles
- triangle measurement 227
- trigonometric equations 635–7
 - involving radians 637–8
- trigonometric functions 624–9
 - applications 639–40
 - centre of 632
 - general 632–3
 - inverse 592, 642–4, 645–50
 - period and amplitude 630–1
 - phase shifts 592, 631–2
 - properties 629–33
 - in terms of radians 628–9
- trigonometric identities 599–603
 - double angles 609–10
 - products to sums and differences 611–13
 - sums and differences of angles 605–8
 - t -formulas 592, 613–15
- trigonometric ratios 226, 227
 - exact 272–3
 - obtuse angles 248, 249
 - unit circle 248–9, 624
- trigonometry
 - applications 240–4
 - origins 227
 - see also* right-angled triangles
- trinomials 4, 21
 - factorising 20–1, 23–4, 25
- triple roots of the equation 317
- true bearings 224, 243–4
- turning point 140, 412
 - at multiple roots on polynomial graphs 318, 472
 - parabola 179–80, 182–3
- uniform probability distribution 660, 667–9
- union (set notation) 488
- unit circle
 - angles at any magnitude 592–6
 - negative angles 596–7
 - trigonometric ratios 248–9, 624
- unordered selections 88, 112
- variables 141
- variance 660, 681
- formula 682–4
- velocity 412, 455, 456, 463
- Venn diagrams 486, 488–9
- vertex (parabola) 140, 179, 182
- vertical line test 140, 143–4
- vertical lines 161–2
- vertically opposite angles 224
- Wallis, John 12, 438
- waves 640
- x -intercept 152
 - $y = a^x$ 537–8
 - $y = \cos^{-1} x$ 643
 - $y = e^x$ 542
 - $y = -f(-x)$, graph of 369–70
 - $y = -f(x)$, graph of 356–7
 - $y = f(-x)$, graph of 367–9
 - $y = f(x)$, graph of 366–70
 - $y = kx^3$, graph of 197
 - $y = \sin^{-1} x$ 642
 - $y = \tan^{-1} x$ 643
 - y -intercept 152
 - zero acceleration 464
 - zero displacement 463
 - zero factorial 99
 - zero indices 7–8
 - zero polynomial 294
 - zero velocity 463
 - zeros of the polynomial 140, 205, 294, 300