

Practice set 3



For Questions 1 to 12, select the correct answer **A**, **B**, **C** or **D**.

- 1 The quotient rule for differentiating $y = \frac{u}{v}$ is:
- A** $y' = \frac{uv' - vu'}{v^2}$ **B** $\frac{u'v - v'u}{v^2}$
C $y' = u'v + v'u$ **D** $y' = uv' + vu'$
- 2 If $f(x) = x^2$ and $g(x) = 2x + 1$, the composite function $g(f(x))$ is given by:
- A** $(2x + 1)^2$ **B** $(2x)^2 + 1$
C $2x + 1^2$ **D** $2x^2 + 1$
- 3 The number of employees, N , is inversely proportional to the time, t , it takes to do a stocktake. What is the equation showing this information?
- A** $N = kt$ **B** $N = t + k$ **C** $N = \frac{k}{t}$ **D** $N = \frac{t}{k}$
- 4 Find the derivative of $(3x - 2)^8$.
- A** $(3x - 2)^7$ **B** $8(3x - 2)^7$
C $8x^7(3x - 2)$ **D** $24(3x - 2)^7$
- 5 Find the probability of drawing out a blue and a white ball from a bag containing 7 blue and 5 white balls if the first ball is not replaced before taking out the second.
- A** $\frac{70}{121}$ **B** $\frac{70}{144}$ **C** $\frac{1225}{17\,424}$ **D** $\frac{35}{66}$
- 6 The equation of a circle with radius 3 and centre $(-1, 4)$ is:
- A** $(x - 1)^2 + (y + 4)^2 = 3$ **B** $(x - 1)^2 + (y + 4)^2 = 9$
C $(x + 1)^2 + (y - 4)^2 = 9$ **D** $(x + 1)^2 + (y - 4)^2 = 3$
- 7 If $f(x) = 2x^2 - 3x + 1$ and $g(x) = (x + 3)^2$ find the degree of $y = f(x)g(x)$.
- A** 2 **B** 4 **C** 3 **D** 5
- 8 Find the domain of $f(x) = \frac{2}{x+7}$.
- A** $(-\infty, 7) \cup (7, \infty)$ **B** $(-\infty, -7) \cup (-7, \infty)$
C $(-\infty, 7) \cap (7, \infty)$ **D** $(-\infty, -7) \cap (-7, \infty)$
- 9 If the displacement of a particle is given by $x = 2t^3 + 6t^2 - 4t + 10$, the initial velocity is:
- A** -4 **B** 10 **C** 12 **D** 14

10 In a group of 25 students, 19 catch a train to school and 21 catch a bus. If one of these students is chosen at random, find the probability that the student only catches a bus to school.

A $\frac{6}{25}$

B $\frac{21}{25}$

C $\frac{3}{5}$

D $\frac{3}{20}$

11 **EXT1** A polynomial equation $P(x) = 0$ has multiplicity 2 at $x = a$. The equation of the polynomial is:

A $P(x) = k(x + a)^2 Q(x)$

B $P(x) = k(x - a)Q(x)$

C $P(x) = k(x + a)Q(x)$

D $P(x) = k(x - a)^2 Q(x)$

12 Conditional probability $P(A|B)$ is given by:

A $\frac{P(A \cup B)}{P(B)}$

B $\frac{P(A \cap B)}{P(A)}$

C $\frac{P(A \cup B)}{P(A)}$

D $\frac{P(A \cap B)}{P(B)}$

13 Differentiate:

a $y = x^9 - 4x^2 + 7x + 3$

b $y = 2x(x^2 - 1)$

c $y = 3x^{-4}$

d $y = \frac{5}{2x^5}$

e $y = \sqrt{x^3}$

f $y = (2x + 3)^7$

g $y = \frac{1}{(x^2 - 7)^4}$

h $y = \sqrt[3]{5x + 1}$

i $y = \frac{5x^2 - 1}{2x + 3}$

14 Sketch the graph of:

a $y = \frac{4}{2x - 4}$

b $P(x) = x^3 + x^2 - 2x$

c $y = |x - 1|$

d $x^2 + y^2 = 25$

e $f(x) = -\sqrt{1 - x^2}$

f **EXT1** $x = 2t, y = 4t^2 - 4$

15 In a class of 25 students, 11 play guitar, 9 play the piano, while 8 don't play either instrument. If one student is selected at random from the class, find the probability that this student will play:

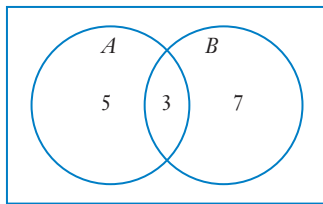
a both guitar and piano

b neither guitar or piano

c only guitar.

- 16** The volume in litres of a rectangular container that is leaking over time t minutes is given by $V = -t^2 + 4t + 100$. Find:
- the initial volume
 - the volume after 10 minutes
 - the rate of change in volume after 10 minutes
 - how long it will take, to 1 decimal place, until the container is empty.
- 17** **a** Find the equation of the tangent to the curve $y = x^3 - 3x$ at the point $P = (-2, -2)$.
b Find the equation of the normal to $y = x^3 - 3x$ at P .
c Find the point Q where this normal cuts the x -axis.
- 18** Two dice are thrown. Find the probability of throwing:
- double 1
 - any double
 - at least one 3
 - a total of 6
 - a total of at least 8.
- 19** The function $f(x) = ax^2 + bx + c$ has a tangent at $(1, -3)$ with a gradient of -1 . It also passes through $(4, 3)$. Find the values of a , b and c .
- 20** Find the equation of the circle with centre $(-2, -3)$ and radius 5 units.
- 21** Find the centre and radius of the circle with equation:
- $x^2 + 6x + y^2 - 10y - 15 = 0$
 - $x^2 + 10x + y^2 - 6y + 30 = 0$
- 22** $f(x) = 3x^2 - 4x + 9$.
- Find $f(x+h) - f(x)$.
 - Show by differentiating from first principles that $f'(x) = 6x - 4$.
- 23** **a** Find the equation of the tangent to the curve $y = x^3 - 2$ at the point $P(1, -1)$.
b The curve $y = x^3 - 2$ meets the y -axis at Q . Find the equation of PQ .
c Find the equation of the normal to $y = x^3 - 2$ at the point $(-1, -3)$.
d Find the point R where this normal cuts the x -axis.
- 24** **EXT1** If $P(x) = (x - 7)^5 Q(x)$, show that $P(7) = P'(7) = 0$.
- 25** 100 cards are numbered from 1 to 100. If one card is chosen at random, find the probability of selecting:
- an even number less than 30
 - an odd number or a number divisible by 9.

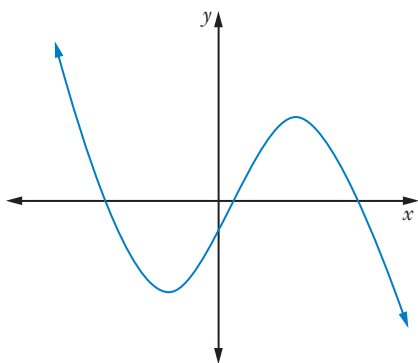
- 26** A bag contains 5 white, 6 yellow and 3 blue balls. Two balls are chosen at random from the bag without replacement. Find the probability of choosing:
- a** 2 blue balls **b** a white ball and a yellow ball
- 27** If Scott buys 10 tickets, find the probability that he wins both first and second prizes in a raffle in which 100 tickets are sold.
- 28** Two dice are rolled. Find the probability of rolling a total:
- a** of 8 **b** less than 7 **c** greater than 9
- d** of 4 or 5 **e** that is an odd number.
- 29** For the Venn diagram, find:



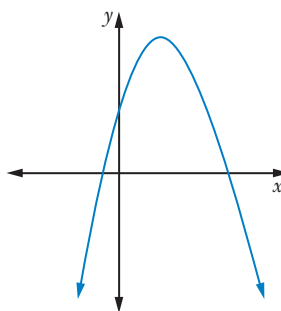
- a** $P(A|B)$ **b** $P(B|A)$
- 30** A bag contains 5 red, 7 blue and 9 yellow balls. Cherylanne chooses 2 balls at random from the bag. Find the probability of that she chooses:
- a** blue given the first ball was yellow
- b** red given the first ball was blue.
- 31** If $f(x) = 2x^3 - 5x^2 + 4x - 1$, find $f(-2)$ and $f'(-2)$.
- 32 a** Find the gradient of the secant to the curve $f(x) = 2x^3 - 7$ between the point (2, 9) and the point where:
- i** $x = 2.01$ **ii** $x = 1.99$
- b** Hence estimate the gradient of the tangent to the curve at (2, 9).

33 Sketch the gradient function for each curve.

a



b



34 The area of a community garden in m^2 is given by $A = 7x - x^2$ where x is the length of the garden.

a Find the area when the length is:

- i** 3 m **ii** 4.5 m.

b Find the length when the area is 8 m^2 , to 1 decimal place.

c Sketch the graph of the area function.

d Find the maximum possible area.

35 Solve graphically:

a $|x + 2| = 3$

b **EXT1** $|3y - 7| \leq 20$

c **EXT1** $\frac{3}{x-4} < 5$

36 **EXT1** The radius of a snowball grows at a constant rate of 0.15 cm s^{-1} as it rolls down a hill. Find the change in its volume when the radius is 8.5 cm.

37 If $f(x) = x^2 - 1$ and $g(x) = x^3 + 3$, find:

a the degree of $y = f(x)g(x)$

b the leading coefficient of $y = f(x)g(x)$

c the constant term of $y = f(x)g(x)$.

38 **EXT1** Find the rate of change in radius of a melting sphere of ice when its radius is 8 cm, if the volume is decreasing at a constant rate of 2.5 cm^3 per second.

39 The displacement x cm of an object moving along a straight line over time t seconds is given by $x = 2t^3 - 13t^2 + 17t + 12$.

a Find the initial displacement, velocity and acceleration.

b Find the displacement, velocity and acceleration after 2 seconds.

c **EXT1** Find the times when the object is at the origin.

d **EXT1** When (correct to one decimal place) is the object moving at constant speed?

- 40** If $A = \{1, 3, 4, 5\}$ and $B = \{2, 3, 5, 6\}$:
- find $A \cup B$
 - find $A \cap B$
 - draw a Venn diagram showing this information.
- 41** Find the equation of the tangent to the curve $y = 3x^2 - 6x + 7$ at the point $(2, 7)$.
- 42** Find the derivative of:
- $y = x^{-3}$
 - $y = \sqrt{x^3}$
 - $y = \frac{1}{x^2}$
 - $y = \frac{(7x+4)^2}{3x-1}$
 - $y = (5x^2 + 1)(2x - 3)^4$
 - $y = (3x + 1)^5$
 - $y = \sqrt{2x-1}$
- 43** **EXT1** Show that if a polynomial $P(x)$ has a double root at $x = 3$, $P(3) = P'(3) = 0$.
- 44** $f(x) = x^2 - 2$ and $g(x) = 2x - 1$.
- Find the equation of:
 - $y = f(x) + g(x)$
 - $y = f(x)g(x)$
 - $y = g(x) - f(x)$
 - $y = \frac{g(x)}{f(x)}$
 - Sketch the graph of:
 - $y = -f(x)$
 - $y = g(-x)$
 - $y = -g(-x)$
 - EXT1** Sketch the graph of:
 - $y = f(x) + g(x)$
 - $y = f(x)g(x)$
 - $y^2 = f(x)$
 - $y = \frac{1}{g(x)}$
 - $y = |f(x)|$
 - $y = g(|x|)$
- 45** **a** Find the centre and radius of the circle $x^2 + 2x + y^2 - 6y - 6 = 0$.
b Find its domain and range.
- 46** **EXT1** If $f(x) = x - 9$ and $g(x) = x^2 + 7$, sketch the graph of:
- $y = f(x) + g(x)$
 - $y = f(|x|)$
 - $y = \frac{1}{f(x)}$
 - $y^2 = f(x)$
- 47** **EXT1** If $f(x) = (x + 1)(x - 5)(x + 3)$, sketch $y^2 = f(x)$ and state the domain and range.
- 48** Find the equation of the normal to the curve $y = x^2 - 4x + 1$ at the point $(3, -2)$.

49 Differentiate:

a $y = 2x^4 - 5x^3 + 3x^2 - x - 4$

b $y = \frac{1}{2x^5}$

c $y = \sqrt{x}$

d $y = (2x - 3)^7$

e $y = 3x^4(2x - 5)^7$

f $y = \frac{5x + 7}{3x - 2}$

50 If $f(x) = x^2 + 1$ and $g(x) = x - 3$:

a find the degree of:

i $f(x) + g(x)$

ii $f(x)g(x)$

b **EXT1** sketch the graph of:

i $y = f(x) + g(x)$

ii $y = |g(x)|$

iii $y = \frac{1}{g(x)}$

iv $y^2 = f(x)$

v $y = f(x)g(x)$

vi $y^2 = g(x)$

51 A coin is tossed and a die thrown. Find the probability of getting:

a a head and a 6

b a tail and an odd number.

52 Find the domain and range of:

a $y = x^3 + 1$

b $y = 1 - x^2$

c $x^2 + 4x + y^2 - 2y - 20 = 0$

d $y = \frac{4}{x + 2}$

53 **EXT1** The monic polynomial equation $P(x) = 0$ has a root of multiplicity 4 at $x = -2$.

a Write an expression for the polynomial $P(x)$.

b Show that $P(-2) = P'(-2) = 0$.

54 If $f(x) = x^3$ and $g(x) = 2x + 5$, find:

a $f(g(x))$

b $g(f(x))$

55 The table below shows the results of an experiment in tossing 2 coins.

Result	Frequency
HH	24
HT	15
TH	38
TT	23

a Add a column for relative frequencies as fractions.

b From the table, find the probability of tossing:

i 2 tails

ii a head and a tail in any order

c What is the theoretical probability of tossing:

i 2 tails?

ii a head and a tail in any order?

- 56** Find the equation of the tangent to the curve $y = x^3 - 7x + 3$ at the point where $x = 2$.
- 57** Find in exact form:
- a** the length of the arc
 - b** the area of the sector
- cut off by an angle of 40° at the centre of a circle with radius 4 cm.
- 58** **EXT1** Mason's wardrobe has 8 pairs of jeans, 12 T-shirts and 5 pairs of shoes. Mason decides to wear a different combination of jeans, T-shirt and shoes every day. How many days will it take for him to wear every possible combination?
- 59** If $f(x) = |x| - 2$ find:
- a** $f(-2)$
 - b** $f(0)$
 - c** $f(m + 1)$
- 60** The probability that Despina passes her first maths test is 64% and the probability that she will pass both the first and second tests is 48%. Find the probability that Despina passes the second test given that she passes the first test.
- 61** If $P(L) = 45\%$, $P(L \cap M) = 5.4\%$ and $P(M) = 12\%$, show that L and M are independent.
- 62** **EXT1** Write each function in parametric form using the equation for x given.
- a** $y = 2x^2 + 6x, x = 8t$
 - b** $3x - 8y + 12 = 0, x = 2t$
 - c** $x^2 + y^2 = 1, x = \cos t$
 - d** $(x + 3)^2 + (y - 1)^2 = 16, x = 4 \cos t - 3$
- 63** Given $P(X) = 0.26$, $P(Y) = 0.15$ and $P(X \cup Y) = 0.371$, show that X and Y are independent.
- 64** **EXT1** Write each set of parametric equations in Cartesian form.
- a** $x = 2p - 3, y = p - 2$
 - b** $x = 3t + 1, y = 6t^2$
 - c** $x = 4 \cos \theta, y = 4 \sin \theta$
 - d** $x = 3 - 2 \sin \theta, y = 2 + 2 \cos \theta$
- 65** State whether events A and B are mutually exclusive if $P(A) = 0.18$, $P(A \cup B) = 0.5$ and $P(B) = 0.32$.