# Practice set 4

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For Questions **1** to **7**, select the correct answer **A**, **B**, **C** or **D**.

**1** Find the amplitude and period of $y = 5 \sin 3x$.

   **A**   Amplitude 3, period 5          **B**   Amplitude 5, period 3

   **C**   Amplitude 5, period $\dfrac{2\pi}{3}$       **D**   Amplitude 3, period $\dfrac{2\pi}{5}$

**2** The table is a discrete probability distribution.

| $x$ | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| $P(X = x)$ | 0.14 | 0.16 | 0.08 | 0.14 | 0.31 | 0.17 |

   Find $P(X \le 4)$.

   **A**   0.38       **B**   0.52       **C**   0.14       **D**   0.62

**3** Find the exact value of $\sin 135° + \cos 120°$.

   **A**   $\dfrac{\sqrt{2} - \sqrt{3}}{2}$         **B**   $\dfrac{\sqrt{2} + 1}{2}$

   **C**   $\dfrac{\sqrt{2} + \sqrt{3}}{2}$         **D**   $\dfrac{\sqrt{2} - 1}{2}$

**4** Which statement is the same as $3^x = 7$? There is more than one answer.

   **A**   $x = \log \dfrac{7}{3}$        **B**   $\log_3 x = 7$

   **C**   $\log_3 7 = x$        **D**   $x = \dfrac{\log 7}{\log 3}$

**5** The derivative of $x^2(2x + 9)^2$ is:

   **A**   $4x(2x + 9)$        **B**   $2x(2x + 9)^2 + 2x^2(2x + 9)$

   **C**   $2x(2x + 9)$        **D**   $2x(2x + 9)^2 + 4x^2(2x + 9)$

**6** `EXT1` The equation for a quantity $Q$ that grows exponentially over time $t$ according to the formula $\dfrac{dQ}{dt} = k(Q - P)$ is given by:

   **A**   $Q = P + Ae^{kt}$        **B**   $Q = k + Ae^{Pt}$

   **C**   $Q = Pe^{kt}$        **D**   $Q = P - Ae^{kt}$

**7** `EXT1` Simplify $\dfrac{2\tan x}{1 - \tan^2 x}$.

   **A**   $\tan^2 x$       **B**   $\cot x$       **C**   $\tan 2x$       **D**   $\cos 2x$

**8** Differentiate:

**a** $y = e^x - x$   **b** $y = 3e^x + 1$   **c** $y = (e^x - 2)^4$

**d** $y = e^x(4x + 1)^3$   **e** $y = \dfrac{e^x}{5x - 2}$   **f** $y = 5e^{7x}$

**9** A function is given by

$$f(x) = \begin{cases} \dfrac{x+1}{8} & \text{for } x = 0, 1, 2 \\[2mm] \dfrac{x-2}{4} & \text{for } x = 3 \end{cases}$$

**a** Find:

**i** $f(0)$   **ii** $f(3)$

**b** Show that $f(x)$ is a probability function.

**10** Find $\log_5 \dfrac{1}{25}$.

**11** The table represents a probability distribution.

| $x$ | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| $P(X = x)$ | $\dfrac{1}{10}$ | $\dfrac{3}{10}$ | $\dfrac{1}{5}$ | $\dfrac{1}{10}$ | $\dfrac{1}{10}$ | $\dfrac{1}{5}$ |

Find:

**a** $P(X = 2)$   **b** $P(X < 4)$   **c** $P(X \geq 2)$

**d** $P(4 \leq X \leq 6)$   **e** $P(1 \leq X < 5)$

**12** Simplify:

**a** $\tan(180° - \theta)$   **b** $\sin(-\theta)$   **c** $\cos(2\pi - \theta)$

**13** For $0 \leq x \leq 2\pi$ sketch the graph of:

**a** $y = 2\sin 4x$   **b** $y = \tan \dfrac{x}{2}$   **c** $y = -\cos x$

**14** **EXT1** Find the exact value of:

**a** $\sin^{-1}\left(\dfrac{1}{2}\right) + \cos^{-1}\left(\dfrac{1}{2}\right)$   **b** $\cos(\tan^{-1} 1)$

**15** For each random variable $X$, write the set of possible values.

**a** The number of rolls of a die until a 6 turns up

**b** The number of red cards selected when choosing 12 cards from a bag containing 15 red and 15 black cards

**c** The first rainy day in January.

**16** Solve $\log_x \dfrac{1}{16} = 4$.

**17** The population of a city over $t$ years is given by the formula $P = 100\ 000e^{0.71t}$. After how many years, to 1 decimal place, will the population become 1 million?

**18** A bag contains 7 white and 6 blue cards. Create a probability distribution table for the number of blue cards selected when randomly selecting 3 cards:

   **a**   with replacement      **b**   without replacement.

**19** If $\tan x = -\dfrac{4}{3}$ and $\cos x > 0$, evaluate $\sin x$ and $\cos x$.

**20** Solve for $0 \le x \le 2\pi$:

   **a**   $2 \cos x + 1 = 0$          **b**   $\tan^2 x = 1$

   **c**   $\cos x = 0$                **d**   $\sin 2x = \dfrac{1}{2}$

**21** This table represents a probability distribution.

| $x$ | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| $P(X = x)$ | 0.16 | 0.23 | 0.22 | $a$ | $b$ |

If $E(X) = 3.04$, evaluate $a$ and $b$.

**22** Find the expected value, variance and standard deviation for the probability distribution below.

| $x$ | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| $P(X = x)$ | 0.2 | 0.1 | 0.3 | 0.1 | 0.3 |

**23** Find the exact value of:

   **a**   $\cos \dfrac{7\pi}{4}$          **b**   $\sin \dfrac{4\pi}{3}$          **c**   $\tan \dfrac{5\pi}{6}$

**24** Draw a discrete probability distribution table for the number of tails when tossing 3 coins.

**25** Sketch the graph of:

   **a**   $y = \log_3 x$                **b**   $y = 3 \log_2 x - 1$

**26** **a**   Write $\log_e x$ as an equation with $x$ in terms of $y$.

   **b**   Hence find the value of $x$, to 3 significant figures, when $y = 1.23$.

**27** Solve $7^{2x} = 3$.

**28** This table shows a discrete probability distribution. Evaluate $k$.

| $x$ | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| $P(X = x)$ | $2k$ | $3k$ | $4k - 2$ | $5k - 1$ | $6k$ |

**29** EXT1 Sketch the graph of the inverse function of $y = \log_2 x$.

**30** State whether each probability distribution is uniform.
   **a** Number of heads when tossing 2 coins
   **b** Number of heads when tossing a coin
   **c** Number of even numbers when rolling one die
   **d** Number of 1s when rolling one die.

**31** State whether each function is a probability function.

   **a** $f(x) = \dfrac{x+1}{10}$ for $x = 0, 1, 2, 3$    **b** $f(x) = \begin{cases} \dfrac{x}{11} & \text{for } x = 1, 2 \\ \dfrac{x-1}{22} & \text{for } x = 3, 4, 5 \end{cases}$

**32** Solve for $0° \leq x \leq 360°$:
   **a** $\tan x = -1$    **b** $2 \sin x = 1$
   **c** $2 \cos^2 x = 1$    **d** $\tan 2x = \sqrt{3}$

**33** Evaluate, to 2 decimal places where appropriate.
   **a** $\log_2 16$    **b** $\log_3 3$    **c** $\log_4 2$
   **d** $\log_{10} 109.7$    **e** $\ln 43.1$    **f** $\log_3 11$

**34** EXT1 Evaluate:
   **a** $\cos^{-1}\left(-\dfrac{\sqrt{3}}{2}\right)$    **b** $\sin^{-1}(-1)$    **c** $\tan^{-1}\left(\dfrac{1}{\sqrt{3}}\right)$
   **d** $\cos(\sin^{-1} 0)$    **e** $\tan^{-1}\left(\tan\dfrac{4\pi}{3}\right)$

**35** EXT1 If 100 g of a substance decays to 80 g after 3 years, find, to 1 decimal place:
   **a** its mass after 10 years
   **b** when it will decay to 50 g (its half-life)
   **c** the rate at which it is decaying after 3 years.

**36** Sketch the graph of:
   **a** $y = e^{-x}$
   **b** $y = 2e^{3x} + 1$

**37** The probability of winning a game is 65% and the probability of losing the game is 12%.

    **a**   Draw a probability distribution table showing 0 for a loss, 1 for a draw and 2 for a win.

    **b**   Find the expected value and variance.

**38** EXT1 Sketch the graph of:

    **a**   $y = \sin^{-1} x$         **b**   $y = 3 \cos^{-1} 2x$         **c**   $y = \tan^{-1} x$

**39** Find the equation of the tangent to the curve $y = 5e^x$ at the point $(2, 5e^2)$.

**40** EXT1 Show that $\cos^{-1}\left(-\dfrac{1}{2}\right) = \pi - \cos^{-1}\left(\dfrac{1}{2}\right)$.

**41** In a game, Faizal pays \$1 to toss 2 coins. He wins \$2 for 2 heads or 2 tails and loses \$1 for a head and a tail.

    **a**   Find the expected value for this game.

    **b**   How much would you expect Faizal to win or lose in the long term?

**42** EXT1 Find the inverse of each function:

    **a**   $f(x) = 2x - 7$         **b**   $f(x) = x^3 + 1$         **c**   $f(x) = e^{3x}$

**43** A spinner has the numbers 1 to 8 equally placed around it.

    **a**   Draw a probability distribution table for the spinner.

    **b**   Is it a uniform distribution?

    **c**   Find the probability of spinning a number:

        **i**   greater than 4         **ii**   3 or less         **iii**   at least 4

    **d**   Find the expected value of the spinner.

**44** **a**   Show that the points (1, 27%), (2, 31%), (3, 28%) and (4, 14%) represent a discrete probability function.

    **b**   Find $E(X)$ and $Var(X)$.

**45** For the following probability distribution, evaluate $k$.

| $x$ | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| $p(x)$ | $\dfrac{5}{16}$ | $k$ | $\dfrac{1}{16}$ | $\dfrac{3}{8}$ | $\dfrac{1}{16}$ | $\dfrac{1}{8}$ |

**46** Simplify:

**a**   $5 + 5\tan^2 x$

**b**   $\dfrac{(1 + \sin x)(1 - \sin x)}{\sin x \cos x}$

**c**   EXT1 $\cos x \cos y - \sin x \sin y$

**d**   EXT1 $\cos^2 B - \sin^2 B$

**e**   EXT1 $\dfrac{1}{2}[\sin(x + y) - \sin(x - y)]$

**47** Find the exact value of:

**a**   $\tan 150°$

**b**   $\cos(-45°)$

**c**   $\sin 240°$

**48** Find the value of $x$:

**a**   $x^2 - 2x - 3 = 0$

**b**   $1 < 2x - 3 \le 7$

**c**   $|3x + 1| = 4$

**d**   EXT1 $x^2 \ge 4$

**e**   EXT1 $|5x - 7| < 3$

**49** Find the centre and radius of the circle $x^2 - 4x + y^2 + 6y - 3 = 0$.

**50** EXT1 If $f(x) = x^3 - 1$ and $g(x) = x^2 + x$, sketch the graph of:

**a**   $y = |g(x)|$

**b**   $y = f(x)g(x)$

**c**   $y^2 = f(x)$

**51** Amanda leaves home and cycles south for 3.6 km. She then turns and cycles for 5.4 km on a bearing of 243°.

**a**   How far is Amanda from her house, to 1 decimal place?

**b**   What is Amanda's bearing from her house, to the nearest degree?

**52** EXT1 In how many ways can the letters of the word TRIGONOMETRY be arranged?

**53** EXT1 Expand:

**a**   $(3x + y)^5$

**b**   $\left(\sqrt{2} - 3\right)^4$

**54** EXT1 Find the remainder when dividing $x^3 + 7x^2 - 3x - 4$ by $x - 2$.

**55** EXT1 The monic polynomial equation $P(x) = 0$ has a root of multiplicity 4 at $x = -2$.

**a**   Write an expression for the polynomial $P(x)$.

**b**   Show that $P(-2) = P'(-2) = 0$.