9.

PROBABILITY

Probability is the study of how likely it is that something will happen. It is used to make predictions and decisions in areas such as business, investment, weather forecasting and insurance. Statistics also is about analysing data to make decisions. Probability and statistics are closely related.

In this chapter we will look at how to find the probability of something happening, using both experimental data and theoretical probability.

CHAPTER OUTLINE

9.01 Set notation and Venn diagrams
9.02 Relative frequency
9.03 Theoretical probability
9.04 Addition rule of probability
9.05 Product rule of probability
9.06 Probability trees
9.07 Conditional probability



IN THIS CHAPTER YOU WILL:

- understand definitions of probability and associated terminology including set notation, events, outcomes and sample space
- identify differences between experimental and theoretical probability and their limitations
- recognise non-mutually exclusive events and use techniques to count outcomes in these cases
- identify conditional probability and calculate probabilities in these cases
- use probability trees, Venn diagrams and the addition and product rules to calculate probabilities

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TERMINOLOGY

complement: The complement of an event is when the event does not occur
conditional probability: The probability that an event A occurs when it is known that another event B has occurred
equally likely outcomes: Outcomes that have the same chance of occurring
independent events: Events where the occurrence of one event does not affect the probability of another event
mutually exclusive events: Events within the same sample space that cannot both occur at the same time; for example, rolling an even number on a die and rolling a 5 on the same die
non-mutually exclusive events: Events within the same sample space that can occur at the same time; for example, rolling a prime number on a die and rolling an odd number
probability tree: A diagram that uses branches to show multi-stage events and sets out the probability on each branch
relative frequency: The frequency of an event relative to the total frequency
sample space: The set of all possible outcomes in an event
set: A collection of distinct objects called elements or members.
For example, set $A = \{1, 2, 3, 4, 5, 6\}$
tree diagram: A diagram that uses branches to show multi-stage events
Venn diagram: A diagram that shows the relationship between 2 or more sets using circles (usually overlapping) drawn inside a rectangle

WS Venn diagrams

WS Venn diagrams matching activity

WS Set operations

Venn diagrams

9.01 Set notation and Venn diagrams

Probability and statistics do not provide exact answers in real life, but they can help in making decisions. Here are some examples of where statistics and probability are used.

- An actuary is a mathematician who looks at statistics and makes decisions for insurance companies. Life expectancy statistics help to decide the cost of life insurance for people of different ages. Statistics about car accidents will help set car insurance premiums.
- Stockbrokers use a chart or formula to predict when to buy and sell shares. This chart is usually based on statistics of past trends.
- A business does a feasibility study in a local area to decide whether to open up a new leisure centre. It uses this data to make a decision based on the likelihood that local people will want to join the centre.

Sample space

An **outcome** is a possible result of a random experiment.

The **sample space** is the **set** of all possible outcomes for an experiment.

An **event** is a set of one or more outcomes.

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EXAMPLE 1

In a survey, a TV show is given a rating from 1 to 10.

a Write down the sample space.

b Give an example of:

i an outcome **ii** an event.

Solution

a The sample space is the set of all possible ratings from 1 to 10.

Sample space = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}

b **i** There are 10 different possible outcomes. One outcome is a rating of 7.

ii One example of an event is 'a rating higher than 6'.

To find the probability of an event happening, we compare the number of ways the event can occur with the total number of possible outcomes (the sample space).

$$\text{Probability of an event} = \frac{\text{Number of ways the event can occur}}{\text{Total number of possible outcomes}}$$

If we call the event E and the sample space S we can write this as:

Probability formula

$$P(E) = \frac{n(E)}{n(S)}$$

where $P(E)$ means the probability of E, $n(E)$ means the number of outcomes in E and $n(S)$ means the number of outcomes in the sample space S, where each **outcome** is **equally likely**.

We usually write a probability as a fraction, but we could also write it as a decimal or percentage.

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EXAMPLE 2

30 people were surveyed on their favourite sport: 11 liked football, 4 liked basketball, 7 liked tennis, 2 liked golf and 6 liked swimming.

Find the probability that any one of these people selected at random will like:

a swimming **b** football **c** golf.

Solution

The size of the sample space $n(S) = 30$ since 30 people were surveyed.

a $P(\text{swimming}) = \frac{6}{30}$
$= \frac{1}{5}$

b $P(\text{football}) = \frac{11}{30}$

c $P(\text{golf}) = \frac{2}{30}$
$= \frac{1}{15}$

Set notation

When working with probabilities, we often use **set notation**.

Union and intersection

$A \cup B$ means A **union** B and is the set of all elements in set A or set B.

$A \cap B$ means A **intersection** B and is the set of all elements that are in **both** sets A and B.

EXAMPLE 3

Set A contains the numbers 3, 7, 12 and 15.
Set B contains the numbers 2, 9, 12, 13 and 17.

a Write sets A and B in set notation.

b Find $A \cup B$ and $A \cap B$.

Solution

a Set $A = \{3, 7, 12, 15\}$.

Set $B = \{2, 9, 12, 13, 17\}$.

b $A \cup B = \{2, 3, 7, 9, 12, 13, 15, 17\}$. It includes all the numbers in either set A or set B.

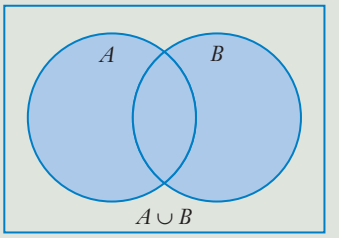
$A \cap B = \{12\}$. It includes any numbers that are in both set A and set B.

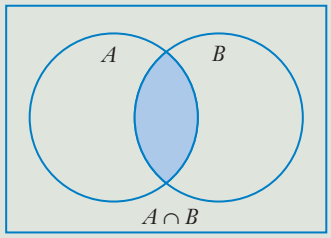
Venn diagrams

A **Venn diagram** is a special way to show the relationship between two or more sets.

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Venn diagram





EXAMPLE 4

Draw a Venn diagram for the integers from 1 to 10, given $A = \{1, 3, 4, 5, 8\}$ and $B = \{3, 6, 8, 9, 10\}$.

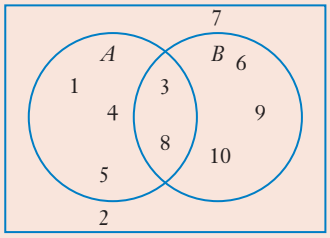
Solution

Draw two overlapping circles and name them A and B.

$A \cap B = \{3, 8\}$, so place these numbers in the overlapping part.

Place the remaining elements of A in the other part of circle A and the remaining elements of B in the other part of circle B.

The numbers are 2 and 7 are not in A or B, so place them outside the circles.



Exercise 9.01 Set notation and Venn diagrams

1 Write the sample space in set notation for each chance situation.

a Tossing a coin

b Rating a radio station between 1 and 5

c Rolling a die

d Selecting a jelly bean from a packet containing red, green, yellow and blue jelly beans

e Rolling an 8-sided die with a different number from 1 to 8 on each face.

2 For each pair of sets, find:

i $X \cap Y$ **ii** $X \cup Y$.

a $X = \{1, 2, 3, 4, 5\}$ and $Y = \{2, 4, 6\}$

b $X = \{\text{red, yellow, white}\}$ and $Y = \{\text{red, white}\}$

c $X = \{4, 5, 7, 11, 15\}$ and $Y = \{6, 8, 9, 10, 12\}$

d $X = \{\text{blue, green, brown, hazel}\}$ and $Y = \{\text{brown, grey, blue}\}$

e $X = \{1, 3, 5, 7, 9\}$ and $Y = \{2, 4, 6, 8, 10\}$

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3 Draw a Venn diagram for each pair of sets.

a $A = \{10, 12, 13, 14, 15\}$ and $B = \{12, 14, 15, 16\}$

b $P = \{\text{red, yellow, white}\}$ and $Q = \{\text{red, green, white}\}$

c $X = \{2, 3, 5, 7, 8\}$ and $Y = \{1, 2, 5, 7, 9, 10\}$

d $A = \{\text{Toyota, Mazda, BMW, Nissan, Porsche}\}$ and $B = \{\text{Mazda, Nissan, Holden, Ford, Porsche}\}$

e $X = \{\text{rectangle, square, trapezium}\}$ and $Y = \{\text{square, parallelogram, trapezium, kite}\}$

4 Discuss whether each probability statement is true.

a The probability of one particular horse winning the Melbourne Cup is $\frac{1}{20}$ if there are 20 horses in the race.

b The probability of a player winning a masters golf tournament is $\frac{1}{15}$ if there are 15 players in the tournament.

c A coin came up tails 8 times in a row. So the next toss must be a head.

d A family has 3 sons and is expecting a fourth child. There is more chance of the new baby being a daughter.

e The probability of a Ducati winning a MotoGP this year is $\frac{6}{47}$ because there are 6 Ducatis and 47 motobikes altogether.

5 To start playing a board game, Simone must roll an even number on a die.

a Write down the sample space for rolling a die.

b What is the set of even numbers on a die?

6 Draw a Venn diagram for each pair of sets.

a Event $K = \{\text{Monday, Thursday, Friday}\}$ and Event $L = \{\text{Tuesday, Thursday, Saturday}\}$ out of days of the week

b Event $A = \{3, 5, 6, 8\}$ and Event $B = \{4, 5, 7, 9\}$ with cards each with a number from 1 to 10 drawn out of a hat

DID YOU KNOW?

John Venn

Venn diagrams are named after **John Venn** (1834–1923), an English probabilist and logician.

WS Experimental probability

9.02 Relative frequency

We can use frequency distribution tables to find the probability of an event using **relative frequency**, the frequency of the event relative to the total frequency.

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EXAMPLE 5

This table shows the number of items bought by a group of people surveyed in a shopping centre.

Number of items	Frequency
0	6
1	4
2	5
3	3
4	7

a Find the relative frequency for each number of items as a fraction.

b Find the probability that a person surveyed at random would buy:

i no items

ii at least 3 items.

Solution

a The sum of the frequencies is 25. This means that 25 people were surveyed.

From the table, 0 has a frequency of 6. The relative frequency is 6 out of 25 $= \frac{6}{25}$.
Similarly, other relative frequencies are:

1 item: $\frac{4}{25}$ 2 items: $\frac{5}{25} = \frac{1}{5}$ 3 items: $\frac{3}{25}$ 4 items: $\frac{7}{25}$

b **i** $P(0) = \frac{6}{25}$

ii At least 3 items means 3 or 4 items. Relative frequency of 3 or 4 is 3 + 7 = 10.

$P(\geq 3) = \frac{10}{25} = \frac{2}{5}$

Exercise 9.02 Relative frequency

1 The table shows the scores that a class earned on a maths test.

Score	Frequency
4	6
5	4
6	1
7	7
8	2
9	3

a Find the relative frequency for each score in the table, in fraction form.

b If a student is chosen at random from this class, find the probability that this student:

i scored 8

ii scored less than 7

iii passed, if the pass mark is 5.

c What score is:

i most likely?

ii least likely?

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2 The table shows the results of a survey into the number of days students study each week.

Number of days	Frequency
1	3
2	6
3	1
4	7
5	2
6	1

a Find the relative frequency as a percentage for each number of days.

b If a student was selected at random, find the most likely number of days this student studies.

c Find the probability that this student would study for:

i 1 day

ii 5 days

iii 3 or 4 days

iv at least 4 days

v fewer than 3 days.

3 The table shows the results of a trial HSC exam.

Class	Frequency
0–19	9
20–39	12
40–59	18
60–79	7
80–99	4

a Calculate the relative frequency as a decimal for each class.

b Find the probability that a student chosen at random from these students scored:

i between 20 and 39

ii between 60 and 99

iii less than 40.

4 This table shows the results of a science experiment to find the velocity of an object when it is rolled down a ramp.

Velocity (m/s)	Frequency
2–4	2
5–7	7
8–10	4
11–13	1
14–16	6

a Write the relative frequency of each velocity as a fraction.

b Find the probability that an object selected at random rolls down the ramp with a velocity between:

i 5 and 7 m/s **ii** 11 and 13 m/s **iii** 8 and 10 m/s

iv 11 and 16 m/s **v** 2 and 10 m/s.

c Find the probability that the object has a velocity:

i less than 8 m/s **ii** 5 m/s or more **iii** more than 7 m/s.

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5 A telemarketing company records the number of sales it makes per minute over a half-hour period. The results are in the table.

Sales/min	Frequency
0	4
1	12
2	6
3	3
4	0
5	5

a What percentage of the time were there 3 sales per minute?

b Write the relative frequencies as percentages.

c What is the most likely number of sales/minute?

d Find the probability of making:

i 2 sales/minute **ii** 5 sales/minute

iii more than 2 sales per minute.

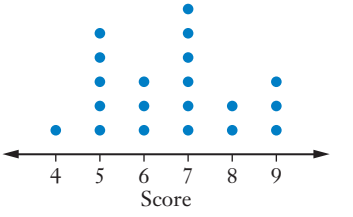
6 a Organise the scores below in a frequency distribution table.

9, 5, 4, 7, 7, 9, 4, 6, 5, 8, 9, 6, 7, 4, 4, 3, 8, 5, 6, 9

b Find the probability of an outcome chosen at random having a score of:

i 7 **ii** at least 8 **iii** less than 5 **iv** 7 or less

7 a From the dot plot, draw up a frequency distribution table.



b Find the probability (as a decimal) that an outcome chosen at random has a score of:

i 8 **ii** at least 6

iii less than 7 **iv** 5 or more

v 8 or less

8 The stem-and-leaf plot shows the ages of people attending a meeting.

Stem	Leaf
1	8 9 9
2	0 3 5 6
3	0 1 2 2 3 5 7 9
4	2 4 6 7 8
5	1 2 4 4 6

a Organise this data into a frequency distribution table, using groups of 10–19, 20–29 and so on.

b What percentage of people at the meeting were:

i in their 30s?

ii younger than 20?

iii in their 40s or 50s?

c Find the probability that a person selected at random from this meeting is:

i younger than 40

ii 50 or over

iii between 20 and 49

iv over 29

v between 10 and 49.

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9 The table shows the quantity of food that a pet shop uses each day for a month.

Food (kg)	Frequency
0–14	3
15–29	11
30–44	8
45–59	4
60–74	2

a In which month was this survey done?

b For what fraction of the month was 45–59 kg of food used?

c For what percentage of the month did the pet shop need more than 29 kg of pet food?

d Find the relative frequency for all groups as a fraction.

e If this survey is typical of the quantities of food that the pet shop uses, find the probability that on any day it will use between:

i 30 and 44 kg **ii** 45 and 74 kg **iii** 0 and 29 kg

iv 15 and 59 kg **v** 30 and 74 kg

WS Theoretical probability

Matching probabilities

9.03 Theoretical probability

While experiments and surveys can give a good prediction of the probability of future events, they are not very accurate. The larger the number of trials, the closer the results can become to the theoretical probability. However, this is not guaranteed.

For example, it is reasonable to assume that if you toss a coin many times you would get similar numbers of heads and tails. Yet in an experiment a coin may come up heads every time.

INVESTIGATION

TOSSING A COIN

Toss a coin 20 times and count the number of heads and tails. What would you expect to happen when tossing a coin this many times? Did your results surprise you?

Combine your results with others in your classroom into a table with relative frequencies.

1 Do the combined results differ from your own?

2 From the table, find the probability of tossing

a heads **b** tails.

If a coin came up tails every time it was tossed 20 times, do you think it would it be more likely to come up heads the next time? Why?

Even though we might think that theoretical probability should be more accurate than experiments, in real life these probabilities will not happen exactly as in theory!

Mutually exclusive events are events that cannot occur at the same time. For example, when throwing a die you cannot throw a number that is both a 5 and a 6.

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The addition rule for mutually exclusive events

When events A and B are mutually exclusive:

$$P(A \cup B) = P(A) + P(B)$$

EXAMPLE 6

A container holds 5 blue, 3 white and 7 yellow marbles. If one marble is selected at random, find the probability of selecting:

a a white marble

b a white or blue marble

c a yellow, white or blue marble

d a red marble.

Solution

Blue, white and yellow are mutually exclusive events.

$n(S) = 5 + 3 + 7 = 15$

a $P(\text{W}) = \frac{3}{15}$

$= \frac{1}{5}$

b $P(\text{W} \cup \text{B}) = P(\text{W}) + P(\text{B})$

$= \frac{3}{15} + \frac{5}{15}$

$= \frac{8}{15}$

c $P(\text{Y} \cup \text{W} \cup \text{B}) = P(\text{Y}) + P(\text{W}) + P(\text{B})$

$= \frac{7}{15} + \frac{3}{15} + \frac{5}{15}$

$= \frac{15}{15}$

$= 1$

d $P(\text{R}) = \frac{0}{15}$

$= 0$

The range of probabilities

If $P(E) = 0$ the event is impossible.

If $P(E) = 1$ the event is certain (it has to happen).

$0 \leq P(E) \leq 1$

The sum of all (mutually exclusive) probabilities is 1.

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Complementary events

The **complement** of set E is the set of all elements that are not in E. We write $\overline{E}$ or E^c.

The **complement** $\overline{E}$ of an event E happening is the event **not** happening.

$P(\overline{E}) = 1 - P(E)$

$P(E) + P(\overline{E}) = 1$

EXAMPLE 7

a The probability of winning a raffle is $\frac{1}{350}$. What is the probability of not winning?

b The probability of a tree surviving a fire is 72%. Find the probability of the tree failing to survive a fire.

Solution

a $P(\text{not win}) = 1 - P(\text{win})$

$= 1 - \frac{1}{350}$

$= \frac{349}{350}$

b $P(\text{failing to survive}) = 1 - P(\text{surviving})$

$= 100\% - 72\%$

$= 28\%$

We can use probability to make predictions or decisions.

EXAMPLE 8

The probability that a traffic light will turn green as a car approaches it is $\frac{5}{12}$. A taxi goes through 192 intersections where there are traffic lights. How many of these would be expected to turn green as the taxi approached?

Solution

It is expected that $\frac{5}{12}$ of the traffic lights would turn green.

$$\frac{5}{12} \times 192 = 80$$

So it would be expected that 80 traffic lights would turn green as the taxi approached.

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Exercise 9.03 Theoretical probability

1 Alannah is in a class of 30 students. If one student is chosen at random to make a speech, find the probability that the student chosen:

a will be Alannah **b** will not be Alannah.

2 A pack of cards contains 52 different cards, one of which is the ace of diamonds. If one card is chosen at random, find the probability that it:

a will be the ace of diamonds **b** will not be the ace of diamonds.

3 There are 6 different newspapers sold at the local newsagent each day. Wendy sends her little brother Rupert to buy her a newspaper one morning but forgets to tell him which one. What is the probability that Rupert will buy the correct newspaper?

4 A raffle is held in which 200 tickets are sold. If I buy 5 tickets, what is the probability of:

a my winning **b** my not winning the prize in the raffle?

5 In a lottery, 200 000 tickets are sold. If Lucia buys 10 tickets, what is the probability of her winning first prize?

6 A bag contains 6 red balls and 8 white balls. If Peter draws one ball out of the bag at random, find the probability that it will be:

a white **b** red.

7 A shoe shop orders in 20 pairs of black, 14 pairs of navy and 3 pairs of brown school shoes. If the boxes are all mixed up, find the probability that one box selected at random will contain brown shoes.

8 The probability of a bus arriving on time is estimated at $\frac{18}{33}$.

a What is the probability that the bus will not arrive on time?

b If there are 352 buses each day, how many would be expected to arrive on time?

9 A bag contains 5 black marbles, 4 yellow marbles and 11 green marbles. Find the probability of drawing 1 marble out at random and getting:

a a green marble **b** a yellow or a green marble.

10 The probability of a certain seed producing a plant with a pink flower is $\frac{7}{9}$.

a Find the probability of the seed producing a flower of a different colour.

b If 189 of these plants are grown, how many of them would be expected to have a pink flower?

11 If a baby has a 0.2% chance of being born with a disability, find the probability of the baby being born without a disability.

12 A die is thrown. Calculate the probability of throwing:

a a 6 **b** an even number **c** a number less than 3.

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13 A book has 124 pages. If any page is selected at random, find the probability of the page number being:

a either 80 or 90 **b** a multiple of 10

c an odd number **d** less than 100.

14 A machine has a 1.5% chance of breaking down at any given time.

a What is the probability of the machine not breaking down?

b If 2600 of these machines are manufactured, how many of them would be expected to:

i break down? **ii** not break down?

15 The probabilities when 3 coins are tossed are as follows:

$P(3 \text{ heads}) = \frac{1}{8}$ $P(2 \text{ heads}) = \frac{3}{8}$

$P(1 \text{ head}) = \frac{3}{8}$ $P(3 \text{ tails}) = \frac{1}{8}$

Find the probability of tossing at least one head.

16 In the game of pool, there are 15 balls, each with the number 1 to 15 on it. In Kelly pool, each person chooses a number at random to determine which ball to sink. If Tracey chooses a number, find the probability that her ball will be:

a an odd number **b** a number less than 8 **c** the 8 ball.

17 **a** Find the probability of a coin coming up heads when tossed.

b If the coin is double-headed, find the probability of tossing a head.

18 A student is chosen at random to write about his or her favourite sport. If 12 students like tennis best, 7 prefer soccer, 3 prefer squash, 5 prefer basketball and 4 prefer swimming, find the probability that the student chosen will write about:

a soccer **b** squash or swimming **c** tennis.

19 There are 29 red, 17 blue, 21 yellow and 19 green chocolate beans in a packet. If Kate chooses one at random, find the probability that it will be red or yellow.

20 The probability of breeding a white budgerigar is $\frac{2}{9}$. If Mr Seed breeds 153 budgerigars over the year, how many would be expected to be white?

21 A biased coin is weighted so that heads comes up twice as often as tails. Find the probability of tossing a tail.

22 A die has the centre dot painted white on the 5 so that it appears as a 4. Find the probability of throwing:

a a 2 **b** a 4 **c** a number less than 5.

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23 The probabilities of a certain number of seeds germinating when 4 seeds are planted are:

Number of seeds	0	1	2	3	4
Probability	$\frac{3}{49}$	$\frac{18}{49}$	$\frac{16}{49}$	$\frac{8}{49}$	$\frac{4}{49}$

Find the probability of at least one seed germinating.

24 The probabilities of 4 friends being chosen for a soccer team are:

$P(4 \text{ chosen}) = \frac{1}{15}$ $\quad$ $P(3 \text{ chosen}) = \frac{4}{15}$

$P(2 \text{ chosen}) = \frac{6}{15}$ $\quad$ $P(1 \text{ chosen}) = \frac{2}{15}$

Find the probability of:

a none of the friends being chosen

b at least 1 of the friends being chosen.

25 If 2 events are mutually exclusive, what could you say about $A \cap B$?

DID YOU KNOW?

The origins of probability

Girolamo Cardano (1501–76) was a doctor and mathematician who developed the first theory of probability. He was a great gambler, and he wrote *De Ludo Aleae* ('On Games of Chance'). This work was largely ignored, and it is said that the first book on probability was written by **Christiaan Huygens** (1629–95).

The main study of probability was done by **Blaise Pascal** (1623–62), and **Pierre de Fermat** (1601–65). Pascal developed the 'arithmetical triangle' you studied in Chapter 3. Pascal's triangle has properties that are applicable to probability as well.

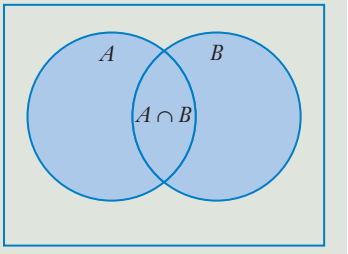
9.04 Addition rule of probability

Sometimes, there is an overlap where more than one event can occur at the same time. We call these **non-mutually exclusive events**. It is important to count the possible outcomes carefully when this happens. We need to be careful not to count the overlapping outcomes $A \cap B$ twice.

Addition rule of probability

For events A and B:

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$



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If A and B are mutually exclusive, then $P(A \cap B) = 0$ so $P(A \cup B) = P(A) + P(B)$.

EXAMPLE 9

One card is selected at random from a pack of 100 cards numbered from 1 to 100. Find the probability that the number on this card is even or less than 20.

Solution

Even: $A = \{2, 4, 6, \ldots, 100\}$

There are 50 even numbers between 1 and 100.

Less than 20: $B = \{1, 2, 3, \ldots, 19\}$

There are 19 numbers less than 20.

Even and less than 20: $A \cap B = \{2, 4, 6, 8, 10, 12, 14, 16, 18\}$

There are 9 numbers that are both even and less than 20.

P(Even or less than 20):

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= \frac{50}{100} + \frac{19}{100} - \frac{9}{100}$$

$$= \frac{60}{100}$$

$$= \frac{3}{5}$$

This is to avoid counting the 9 'overlapping' numbers twice.

Sometimes for more complex problems, a Venn diagram is useful.

EXAMPLE 10

In Year 7 at Mt Random High School, every student must do art or music. In a group of 100 students surveyed, 47 do music and 59 do art. If one student is chosen at random from Year 7, find the probability that this student does:

a both art and music **b** only art **c** only music.

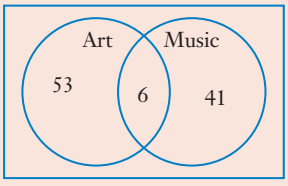
Solution

Number of students: $47 + 59 = 106$

But there are only 100 students!

This means 6 students have been counted twice.

That is, 6 students do both art and music.



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Students doing art only: $59 - 6 = 53$

Students doing music only: $47 - 6 = 41$

a $P(\text{both}) = \frac{6}{100} = \frac{3}{50}$ **b** $P(\text{art only}) = \frac{53}{100}$ **c** $P(\text{music only}) = \frac{41}{100}$

Exercise 9.04 Addition rule of probability

1 A number is chosen at random from the numbers 1 to 20. Find the probability that the number chosen will be:

a divisible by 3 **b** less than 10 or divisible by 3

c a composite number **d** a composite number or a number greater than 12.

2 A set of 50 cards is labelled from 1 to 50. One card is drawn out at random. Find the probability that the card will be:

a a multiple of 5

b an odd number

c a multiple of 5 or an odd number

d a number greater than 40 or an even number

e less than 20.

3 A set of 26 cards, each with a different letter of the alphabet on it, is placed in a box and one card is drawn out at random. Find the probability that the letter on the card is:

a a vowel **b** a vowel or one of the letters in the word 'random'

c a consonant or one of the letters in the word 'movies'.

4 A set of discs is numbered 1 to 100 and one is chosen at random. Find the probability that the number on the disc will be:

a less than 30 **b** an odd number or a number greater than 70

c divisible by 5 or less than 20.

5 In Lotto, a machine holds 45 balls, each with a different number between 1 and 45 on it. The machine draws out one ball at a time at random. Find the probability that the first ball drawn out will be:

a less than 10 or an even number

b between 1 and 15 inclusive, or divisible by 6

c greater than 30 or an odd number.

6 A class of 28 students puts on a concert with all class members performing. If 15 dance and 19 sing in the performance, find the probability that any one student chosen at random from the class will:

a both sing and dance **b** only sing **c** only dance.

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7 A survey of 80 people with dark hair or brown eyes showed that 63 had dark hair and 59 had brown eyes. Find the probability that one of the people surveyed chosen at random has:

- **a** dark hair but not brown eyes
- **b** brown eyes but not dark hair
- **c** both brown eyes and dark hair.

8 A list is made up of 30 people with experience in coding or graphical design. On the list, 13 have coding experience while 9 have graphical design experience. Find the probability that a person chosen at random from the list will have experience in:

- **a** both coding and graphical design
- **b** coding only
- **c** graphical design only.

9 Of a group of 75 students, all study either history or geography. Altogether 54 take history and 31 take geography. Find the probability that a student selected at random studies:

- **a** only geography
- **b** both history and geography
- **c** history but not geography.

10 In a group of 20 dogs at obedience school, 14 dogs will walk to heel and 12 will stay when told. All dogs will do one or the other, or both. If one dog is chosen at random, find the probability that it will:

- **a** both walk to heel and stay
- **b** walk to heel but not stay
- **c** stay but not walk to heel.



Multi-stage problems

9.05 Product rule of probability

CLASS DISCUSSION

TWO-STAGE EVENTS

Work in pairs and try these experiments with one person doing the activity and one recording the results. Toss two coins as many times as you can in a 5-minute period and record the results in a table:

Result	2 heads	One head and one tail	2 tails
Tally			

Compare your results with others in the class. What do you notice? Is this surprising?

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Roll 2 dice as many times as you can in a 5-minute period, find the total of the 2 numbers rolled and record the results in a table:

Total	2	3	4	5	6	7	8	9	10	11	12
Tally											

Compare your results with others in the class. What do you notice? Is this surprising?

Tossing 2 coins and rolling 2 dice are examples of **multi-stage experiments**, where two outcomes happen together. The sample space becomes more complicated, so to list all possible outcomes we use tables and **tree diagrams**.

EXAMPLE 11

Find the sample space and the probability of each outcome for:

a tossing 2 coins

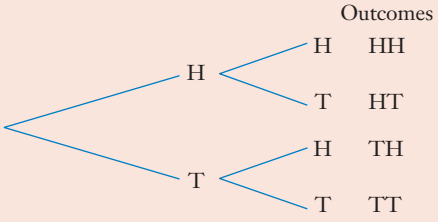
b rolling 2 dice and calculating their sum.

Solution

a Using a table gives:

		2nd coin	
		H	**T**
1st coin	**H**	HH	HT
	T	TH	TT

Using a tree diagram gives:



Since there are four possible outcomes (HH, HT, TH, TT), each outcome has a probability of $\frac{1}{4}$.

Remember that each outcome when tossing 1 coin is $\frac{1}{2}$.

Notice that $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$.

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b A tree diagram would be too big to draw for this question.

Using a table:

		2nd die					
		1	**2**	**3**	**4**	**5**	**6**
1st die	**1**	2	3	4	5	6	7
	2	3	4	5	6	7	8
	3	4	5	6	7	8	9
	4	5	6	7	8	9	10
	5	6	7	8	9	10	11
	6	7	8	9	10	11	12

Since there are 36 outcomes, each has a probability of $\frac{1}{36}$.

Remember that each outcome when rolling 1 die is $\frac{1}{6}$.

Notice that $\frac{1}{6} \times \frac{1}{6} = \frac{1}{36}$.

If A and B are **independent events**, then A occurring does not affect the probability of B occurring. The probability of both occurring is the product of their probabilities.

The product rule for independent events

$$P(A \cap B) = P(A)P(B)$$

EXAMPLE 12

a Find the probability of rolling a double 6 on 2 dice.

b The probability that an archer will hit a target is $\frac{7}{8}$. Find the probability that the archer will:

i hit the target twice

ii miss the target twice.

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Solution

a $P(A \cap B) = P(A)P(B)$

$P(6 \cap 6) = P(6)P(6)$

$= \frac{1}{6} \times \frac{1}{6}$

$= \frac{1}{36}$

b **i** $P(A \cap B) = P(A)P(B)$

Let H = hit, M = miss

$P(\text{H} \cap \text{H}) = P(\text{H})P(\text{H})$

$= \frac{7}{8} \times \frac{7}{8}$

$= \frac{49}{64}$

ii $P(\text{M}) = P(\bar{\text{H}})$

$= 1 - \frac{7}{8}$

$= \frac{1}{8}$

$P(\text{M} \cap \text{M}) = P(\text{M})P(\text{M})$

$= \frac{1}{8} \times \frac{1}{8}$

$= \frac{1}{64}$

The sample space changes when events are not independent. The second event is **conditional** on the first event.

EXAMPLE 13

Maryam buys 5 tickets in a raffle in which 95 tickets are sold altogether. There are 2 prizes in the raffle. What is the probability of her winning:

a both first and second prizes?

b neither prize?

c at least one of the prizes?

Solution

a Probability of winning first prize: $P(\text{W}_1) = \frac{5}{95}$

After winning first prize she has 4 tickets left in the raffle out of a total of 94 tickets left.

Probability of winning second prize: $P(\text{W}_2) = \frac{4}{94}$

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Probability of winning both prizes:

$$P(W_1 \cap W_2) = \frac{5}{95} \times \frac{4}{94}$$
$$= \frac{20}{8930}$$
$$= \frac{2}{893}$$

b Probability of not winning first prize:

$$P(\overline{W}_1) = 1 - \frac{5}{95}$$
$$= \frac{90}{95}$$
$$= \frac{18}{19}$$

After not winning first prize, Maryam's 5 tickets are all left in the draw, but the winning ticket is taken out, leaving 94 tickets in the raffle.

Probability of winning second prize: $P(W_2) = \frac{5}{94}$

Probability of not winning second prize:

$$P(\overline{W}_2) = 1 - \frac{5}{94}$$
$$= \frac{89}{94}$$

Probability of winning neither prize:

$$P(\overline{W}_1 \cap \overline{W}_2) = P(\overline{W}_1)P(\overline{W}_2)$$
$$= \frac{90}{95} \times \frac{89}{94}$$
$$= \frac{8010}{8930}$$
$$= \frac{801}{893}$$

c Probability of at least one win:

'at least one' is the same as 'not none'.

$$P(\geq 1 \text{ win}) = 1 - P(0 \text{ wins})$$
$$= 1 - \frac{801}{893} \quad \text{from } \mathbf{b}$$
$$= \frac{92}{893}$$

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Exercise 9.05 Product rule of probability

1 Find the probability of getting 2 heads if a coin is tossed twice.

2 A coin is tossed 3 times. Find the probability of tossing 3 tails.

3 A family has 2 children. What is the probability that they are both girls?

4 A box contains 2 black balls, 5 red balls and 4 green balls. If I draw out 2 balls at random, replacing the first before drawing out the second, find the probability that they will both be red.

5 The probability of a conveyor belt in a factory breaking down at any one time is 0.21. If the factory has 2 conveyor belts, find the probability that at any one time:

a both conveyor belts will break down

b neither conveyor belt will break down.

6 The probability of a certain plant flowering is 93%. If a nursery has 3 of these plants, find the probability that they will all flower.

7 An archery student has a 69% chance of hitting a target. If she fires 3 arrows at a target, find the probability that she will hit the target each time.

8 The probability of a pair of small parrots breeding an albino bird is $\frac{2}{33}$. If they lay 3 eggs, find the probability of the pair:

a not breeding any albinos **b** having all 3 albinos

c breeding at least one albino.

9 A photocopier has a paper jam on average around once every 2400 sheets of paper.

a What is the probability that a particular sheet of paper will jam?

b What is the probability that 2 particular sheets of paper will jam?

c What is the probability that 2 particular sheets of paper will both not jam?

10 In the game Yahtzee, 5 dice are rolled. Find the probability of rolling:

a five 6s **b** no 6s **c** at least one 6.

11 The probability of a faulty computer part being manufactured at Omikron Computer Factory is $\frac{3}{5000}$. If 2 computer parts are examined, find the probability that:

a both are faulty **b** neither is faulty

c at least one is faulty.

12 A set of 10 cards is numbered 1 to 10 and 2 cards are drawn out at random with replacement. Find the probability that the numbers on both cards are:

a odd numbers **b** divisible by 3

c less than 4.

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13 The probability of an arrow hitting a target is 85%. If 3 arrows are shot, find the probability as a percentage, correct to 2 decimal places, of:

a all arrows hitting the target

b no arrows hitting the target

c at least one arrow hitting the target.

14 A coin is tossed n times. Find the probability in terms of n of tossing:

a no tails **b** at least one tail.

15 A bag contains 8 yellow and 6 green lollies. If I choose 2 lollies at random, find the probability that they will both be green:

a if I replace the first lolly before selecting the second

b if I don't replace the first lolly.

16 Mala buys 10 tickets in a raffle in which 250 tickets are sold. Find the probability that she wins both first and second prizes.

17 Two cards are drawn from a deck of 20 red and 25 blue cards (without replacement). Find the probability that they will both be red.

18 A bag contains 100 cards numbered 1 to 100. Scott draws 2 cards out of the bag. Find the probability that:

a both cards are less than 10

b both cards are even

c neither card is a multiple of 5.

19 A box of pegs contains 23 green pegs and 19 red pegs. If 2 pegs are taken out of the box at random, find the probability that both will be:

a green **b** red

20 Find the probability of selecting 2 apples at random from a fruit bowl that contains 8 apples, 9 oranges and 3 peaches.



Tree diagrams


Tree diagrams


Tree diagrams

9.06 Probability trees

A **probability tree** is a tree diagram that shows the probabilities on the branches.

Probability trees

Use the product rule along the branches to find $P(A \cap B)$, the probability of 'A and B'.

Use the addition rule for different branches to find $P(A \cup B)$, the probability of 'A or B'.

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EXAMPLE 14



Probability trees

a Robert has a chance of 0.2 of winning a prize in a Taekwondo competition. If he enters 3 competitions, find the probability of his winning:

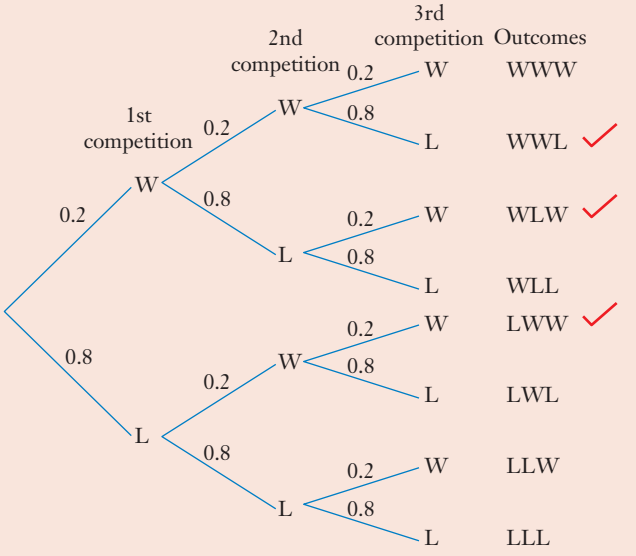
i 2 competitions **ii** at least 1 competition.

b A bag contains 3 red, 4 white and 7 blue marbles. Two marbles are drawn at random from the bag without replacement. Find the probability that the marbles are red and white.

Solution

a $P(\text{W}) = 0.2$, $P(\text{L}) = 1 - 0.2 = 0.8$. W = win, L = lose

Draw a probability tree with 3 levels of branches as shown.



i There are 3 different ways of winning 2 competitions (WWL, WLW and LWW, shown by the red ticks).

Using the product rule along the branches:

$P(\text{WWL}) = 0.2 \times 0.2 \times 0.8 = 0.032$ 'P(win and win and lose)'

$P(\text{WLW}) = 0.2 \times 0.8 \times 0.2 = 0.032$

$P(\text{LWW}) = 0.8 \times 0.2 \times 0.2 = 0.032$

Using the addition rule for the different results:

$P(2 \text{ wins}) = P(\text{WWL}) + P(\text{WLW}) + P(\text{LWW})$ 'P(WWL or WLW or LWW)'

$= 0.032 + 0.032 + 0.032$

$= 0.096$

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ii $P(\geq 1\text{W}) = 1 - P(\text{LLL})$

$= 1 - 0.8 \times 0.8 \times 0.8$

$= 0.488$

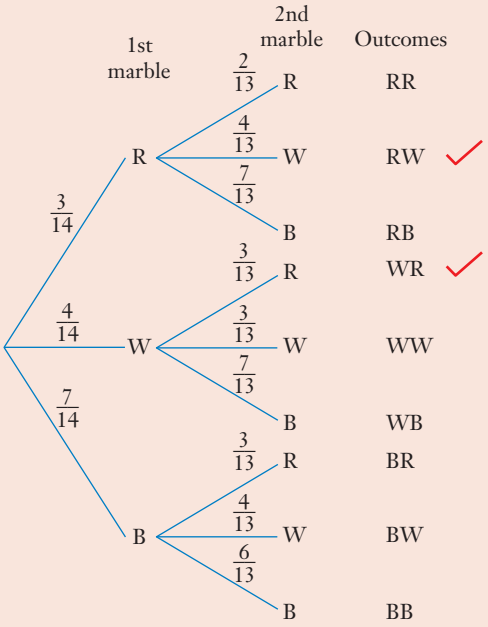
b First marble:

$P(\text{R}) = \frac{3}{14}$ $\quad P(\text{W}) = \frac{4}{14}$

$P(\text{B}) = \frac{7}{14}$



The probabilities for the second marble are dependent on the outcome of the first draw.



There are 2 different ways of drawing out a red and a white marble, as shown by the red ticks: RW, WR.

Using the product rule along the branches:

$P(\text{RW}) = \frac{3}{14} \times \frac{4}{13}$

$= \frac{12}{182}$

$= \frac{6}{91}$

$P(\text{WR}) = \frac{4}{14} \times \frac{3}{13}$

$= \frac{12}{182}$

$= \frac{6}{91}$

Using the addition rule for the different results:

$P(\text{RW or WR}) = \frac{6}{91} + \frac{6}{91}$

$= \frac{12}{91}$

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Exercise 9.06 Probability trees

1 Three coins are tossed. Find the probability of getting:

a 3 tails **b** 2 heads and 1 tail **c** at least 1 head.

2 In a set of 30 cards, each one has a number on it from 1 to 30. If 1 card is drawn out, then replaced and another drawn out, find the probability of getting:

a two 8s

b a 3 on the first card and an 18 on the second card

c a 3 on one card and an 18 on the other card.

3 A bag contains 5 red marbles and 8 blue marbles. If 2 marbles are chosen at random, with the first replaced before the second is drawn out, find the probability of getting:

a 2 red marbles **b** a red and a blue marble.

4 A certain breed of cat has a 35% probability of producing a white kitten. If a cat has 3 kittens, find the probability that she will produce:

a no white kittens **b** 2 white kittens **c** at least 1 white kitten.

5 The probability of rain on any day in May each year is given by $\frac{3}{10}$. A school holds a fete on a Sunday in May for 3 years running. Find the probability that it will rain:

a during 2 of the fetes **b** during 1 fete **c** during least 1 fete.

6 A certain type of plant has a probability of 0.85 of producing a variegated leaf. If I grow 3 of these plants, find the probability of getting a variegated leaf in:

a 2 of the plants **b** none of the plants **c** at least 1 plant.

7 A bag contains 3 yellow balls, 4 pink balls and 2 black balls. If 2 balls are chosen at random, find the probability of getting a yellow and a black ball:

a with replacement **b** without replacement.

8 Anh buys 4 tickets in a raffle in which 100 tickets are sold altogether. There are 2 prizes in the raffle. Find the probability that Anh will win:

a first prize **b** both prizes **c** 1 prize

d no prizes **e** at least 1 prize.

9 Two singers are selected at random to compete against each other in a TV singing contest. One person is chosen from Team A, which has 8 females and 7 males, and the other is chosen from Team B, which has 6 females and 9 males. Find the probability of choosing:

a 2 females **b** 1 female and 1 male.

10 Two tennis players are said to have a probability of $\frac{2}{5}$ and $\frac{3}{4}$ respectively of winning a tournament. Find the probability that:

a 1 of them will win **b** neither one will win.

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11 In a batch of 100 cars, past experience would suggest that 3 could be faulty. If 3 cars are selected at random, find the probability that:

a 1 is faulty **b** none is faulty **c** all 3 cars are faulty.

12 In a certain poll, 46% of people surveyed liked the current government, 42% liked the Opposition and 12% had no preference. If 2 people from the survey are selected at random, find the probability that:

a both will prefer the Opposition

b one will prefer the government and the other will have no preference

c both will prefer the government.

13 A manufacturer of X energy drink surveyed a group of people and found that 31 people liked X drinks best, 19 liked another brand better and 5 did not drink energy drinks. If any 2 people are selected at random from that group, find the probability that:

a one person likes the X brand of energy drink

b both people do not drink energy drinks.

14 In a group of people, 32 are Australian-born, 12 were born in Asia and 7 were born in Europe. If 2 of the people are selected at random, find the probability that:

a they were both born in Asia

b at least 1 of them will be Australian-born

c both were born in Europe.

15 There are 34 men and 32 women at a party. Of these, 13 men and 19 women are married. If 2 people are chosen at random, find the probability that:

a both will be men

b one will be a married woman and the other an unmarried man

c both will be married.

16 Frankie rolls 3 dice. Find the probability she rolls:

a 3 sixes **b** 2 sixes **c** at least 1 six.

17 A set of 5 cards, each labelled with one of the letters A, B, C, D and E, is placed in a hat and 2 cards are selected at random without replacement. Find the probability of getting:

a D and E

b neither D nor E on either card

c at least one D.

18 The ratio of girls to boys at a school is 4 : 5. Two students are surveyed at random from the school. Find the probability that the students are:

a both boys **b** a girl and a boy **c** at least one girl.

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9.07 Conditional probability

Conditional probability is the probability that an event A occurs when it is known that another event B has already occurred. You have already used conditional probability in multi-stage events when the outcome of the second event was dependent on the outcome of the first event. Examples include selections **without replacement**.

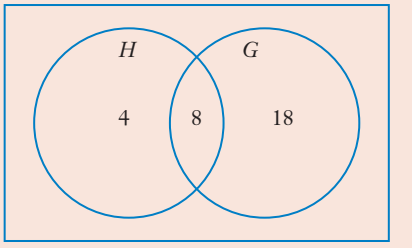
We write the probability of event A happening given that event B has happened as $P(A|B)$.

EXAMPLE 15

All 30 students in a class study either history or geography. If 18 only do geography and 8 do both subjects, find the probability that a student does geography, given that the student does history.

Solution

Draw a Venn diagram using H = history and G = geography.



8 students do both history and geography.

18 students only do geography.

So $n(G) = 18 + 8 = 26$.

So $n(H \text{ only}) = 30 - 26 = 4$

There are $8 + 4 = 12$ students doing history, of whom 8 also do geography.

$$\text{So } P(G|H) = \frac{8}{12}$$
$$= \frac{2}{3}$$

With conditional probability, knowing that an event has already occurred reduces the sample space. In the example above, the sample space changed from 30 to 12.

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EXAMPLE 16

The table shows the results of a survey into vaccinations against a new virus.

	Vaccinated	Not vaccinated	Totals
Infected	13	159	172
Not infected	227	38	265
Totals	240	197	437

Find the probability that a person selected at random is:

a not vaccinated

b infected given that the person is vaccinated

c not infected given that the person is not vaccinated

d vaccinated given that the person is infected.

Solution

a $n(S) = 437$, $n(\text{not vaccinated}) = 197$

$$P(\text{not vaccinated}) = \frac{197}{437}$$

b $n(\text{vaccinated}) = 240$

$n(\text{infected} \mid \text{vaccinated}) = 13$

$$P(\text{infected} \mid \text{vaccinated}) = \frac{13}{240}$$

c $n(\text{not vaccinated}) = 197$

$n(\text{not infected} \mid \text{not vaccinated}) = 38$

$$P(\text{not infected} \mid \text{not vaccinated}) = \frac{38}{197}$$

d $n(\text{infected}) = 172$

$n(\text{vaccinated} \mid \text{infected}) = 13$

$$P(\text{vaccinated} \mid \text{infected}) = \frac{13}{172}$$

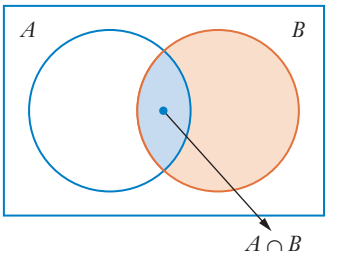
Notice that $P(\text{infected} \mid \text{vaccinated}) \neq P(\text{vaccinated} \mid \text{infected})$.

Conditional probability

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

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The $P(B)$ in the denominator is a result of the sample space being reduced to B (the orange circle in the Venn diagram).



Proof

$$P(A|B) = \frac{n(A \cap B)}{n(B)}$$
$$= \frac{\frac{n(A \cap B)}{n(S)}}{\frac{n(B)}{n(S)}}$$
$$= \frac{P(A \cap B)}{P(B)}$$

For conditional probability, the product rule becomes $P(A \cap B) = P(A|B)P(B)$.

EXAMPLE 17

Lara is an athlete who enters a swimming and running race. She has a 44% chance of winning the swimming race and a 37% chance of winning both races. Find to the nearest whole percentage the probability that she wins the running race if she has won the swimming race.

Solution

If Lara has already won the swimming race (S), then the probability of her winning the running race (R) is conditional.

$$P(S) = 44\%$$
$$= 0.44$$
$$P(R \text{ and } S) = P(R \cap S)$$
$$= 37\%$$
$$= 0.37$$
$$P(R|S) = \frac{P(R \cap S)}{P(S)}$$
$$= \frac{0.37}{0.44}$$
$$\approx 0.84\,09...$$
$$\approx 84\%$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

So the probability of Lara winning the running race given that she has won the swimming race is 84%.

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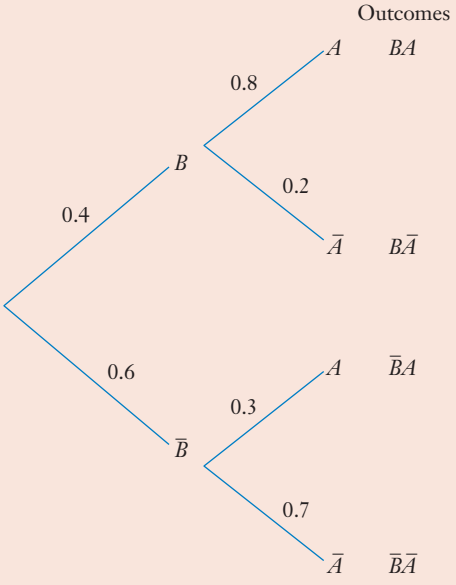
EXAMPLE 18

A zoo has a probability of 80% of having an article published in the newspaper when there is a birth of a baby animal. When there is no birth, the zoo has a probability of only 30% of having an article published. The probability of the zoo having an animal born at any one time is 40%.

Find the percentage probability that a baby animal was born given that an article was published.

Solution

We can draw up a probability tree showing the probabilities of having an article published (A) and a baby animal being born (B).



We want $P(B|A)$. According to the formula:

$$P(B|A) = \frac{P(B \cap A)}{P(A)}$$

From the probability tree:

$P(B \cap A) = P(BA)$ The event (numerator)

$= 0.4 \times 0.8$

$= 0.32$

$P(A) = 0.4 \times 0.8 + 0.6 \times 0.3$ The sample space (denominator)

$= 0.5$

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$$P(B|A) = \frac{P(B \cap A)}{P(A)}$$
$$= \frac{0.32}{0.5}$$
$$= 0.64$$
$$= 64\%$$

So the probability that an animal was born given that an article was published is 64%.

Conditional probability and independent events

We saw earlier that:

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Rearranging this gives $P(A \cap B) = P(A|B)\ P(B)$.

But if A and B are **independent events**, $P(A \cap B) = P(A)P(B)$ (the product rule), which means:

$P(A|B) = P(A)$.

Similarly, $P(B|A) = P(B)$.

Conditional probability and independent events

For independent events A and B:

$$P(A|B) = P(A)$$
$$P(B|A) = P(B)$$
$$P(A \cap B) = P(A)P(B)$$

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EXAMPLE 19

a $P(X) = 0.2$ and $P(X \cap Y) = 0.06$. Determine whether X and Y are independent if:

i $P(Y) = 0.6$ **ii** $P(Y) = 0.3$

b Show that A and B are independent given that $P(A) = 0.6$, $P(B) = 0.45$, $P(A \cup B) = 0.78$.

Solution

a For independent events, the product rule is $P(X \cap Y) = P(X)P(Y)$.

i $P(X \cap Y) = 0.06$

$P(X)P(Y) = 0.2 \times 0.6$

$= 0.12$

$\neq P(X \cap Y)$

$\therefore X$ and Y are not independent.

ii $P(X \cap Y) = 0.06$

$P(X)P(Y) = 0.2 \times 0.3$

$= 0.06$

$= P(X \cap Y)$

$\therefore X$ and Y are independent

b Using the addition rule:

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$0.78 = 0.6 + 0.45 - P(A \cap B)$

$0.78 = 1.05 - P(A \cap B)$

$P(A \cap B) = 1.05 - 0.78$

$= 0.27$

Using the product rule for independent events:

$P(A \cap B) = P(A)\,P(B)$

$= 0.6 \times 0.45$

$= 0.27$, as above

$\therefore A$ and B are independent.

Exercise 9.07 Conditional probability

1 A bag contains 9 black and 8 white balls. I draw out two at random. If the first ball is white, find the probability that the next ball is:

a black **b** white

2 A class has 13 boys and 15 girls. Two students are chosen at random to carry a box of equipment. Find the probability that the second person chosen is a boy given that the first student chosen was a girl.

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3 Two dice are rolled. Find the probability of rolling:

a a double six if the first die was a six.

b a total of 8 or more if the first die was a 3.

4 A team has a probability of 52% of winning its first season and a 39% chance of winning both seasons 1 and 2. What is the probability of the team winning the second season given that it wins the first season?

5 A missile has a probability of 0.75 of hitting a target. It has a probability of 0.65 of hitting two targets in a row. What is the probability that the missile will hit the second target given that it has hit the first target?

6 Danuta has an 80% probability of passing her first English assessment and she has a 45% probability of passing both the first and second assessments. Find the probability that Danuta will pass the second assessment given that she passes the first one.

7 A group of 10 friends all prepared to go out in the sun by putting on either sunscreen or a hat. If 5 put on only sunscreen and 3 put on both sunscreen and a hat, find the probability that a friend who:

a put on sunscreen also put on a hat

b put on a hat didn't put on sunscreen.

8 A container holds 20 cards numbered 1 to 20. Two cards are selected at random. Find the probability that the second card is:

a an odd number given that the first card was a 7

b a number less than 5 given that the first card was a 12

c a number divisible by 3 if the first number was 6.

9 A group of 12 people met at a café for lunch. If 9 people had a pie and 7 had chips, find the probability that one of the people:

a had chips, given that this person had a pie

b did not have a pie given that the person had chips.

10 All except for 3 people out of 25 on a European tour had studied either French or Spanish. Nine people studied only French and 5 studied both French and Spanish. Find the probability that one of these people:

a studied Spanish if that person studied French

b did not study French given that the person studied Spanish.

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11 The two-way table shows the numbers of students who own smartphones and tablets.

	Smartphone	No smartphone	Totals
Tablet	23	8	31
No tablet	65	3	68
Totals	88	11	99

Find the probability that a person selected at random:

a owns a smartphone given that the person:

i owns a tablet **ii** doesn't own a tablet.

b owns a tablet given that the person:

i owns a smartphone **ii** doesn't own a smartphone.

12 The table below shows the number of local people with casual and permanent jobs.

	Women	Men
Permanent	23	38
Casual	79	64

Find the probability that a person chosen at random:

a has a permanent job given that she is a woman

b has a casual job given that he is a man

c is a man given that the person has a casual job

d is a man if the person has a permanent job.

13 In a group of 35 friends, all either play sport or a musical instrument. If 14 play both and 8 only play sport, find the probability that a friend chosen at random will:

a play a musical instrument given that the friend plays sport

b not play sport given that the friend plays a musical instrument.

14 The two-way table shows the results of a survey into attendance at a local TAFE college.

	Under 25	Between 25 and 50	Over 50	Totals
At TAFE	53	68	34	155
Not at TAFE	85	105	88	278
Totals	138	173	122	433

Find the probability that a person:

a attends TAFE given that this person is over 50

b is between 25 and 50 if that person does not attend TAFE

c is not at TAFE given that this person is under 25

d is over 50 if the person is at TAFE

e is at TAFE given the person is aged 25 or over.

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15 A tennis team has a probability of 76% of winning a match when they are at home and 45% of winning a match when they are away. If the team plays 58% of their matches away, find the probability that the team:

- **a** wins their match given that they are away
- **b** are at home given that they win a match
- **c** are away given that they lose a match.

16 A factory produces solar batteries. The probability of a new battery being defective is 3%. However, if the manager is on duty, the probability of a new battery being defective changes to 2%. The manager is on duty 39% of the time. Find the probability that the manager is on duty if a new battery is defective.

17 The chance of a bushfire is 85% after a period of no rain and 21% after rain. The chance of rain is 46%. Find the probability that:

- **a** there is not a bushfire given that it has rained
- **b** it has rained given that there is a bushfire
- **c** it has not rained given there is a bushfire
- **d** it has rained given there is not a bushfire.

18 If $P(A|B) = 0.67$ and $P(B) = 0.31$, find the value of $P(A \cap B)$.

19 If $P(L) = 0.17$, $P(L \cap M) = 0.0204$ and $P(M) = 0.12$, show that L and M are independent.

20 Given $P(X) = 0.3$, $P(Y) = 0.42$ and $P(X \cup Y) = 0.594$, show that X and Y are independent.

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9. TEST YOURSELF

Practice quiz

Probability crossword

Probability find-a-word

For Questions **1**–**4**, select the correct answer **A**, **B**, **C** or **D**.

1 The probability of getting at least one 1 when rolling two dice is:

A $\frac{1}{3}$ **B** $\frac{1}{6}$ **C** $\frac{11}{36}$ **D** $\frac{5}{18}$

2 A bag contains 7 white and 5 blue balls. Two balls are selected at random without replacement. The probability of selecting a white and a blue ball is:

A $\frac{35}{132}$ **B** $\frac{35}{72}$ **C** $\frac{35}{144}$ **D** $\frac{35}{66}$

3 If $A = \{5, 7, 8\}$ and $B = \{3, 7, 9\}$ then the set $\{7\}$ represents:

A $A - B$ **B** $A \cap B$ **C** $A + B$ **D** $A \cup B$

4 For the table, the relative frequency of a score of 11 is (there may be more than one answer):

A 32% **B** $\frac{8}{11}$

C 0.032 **D** $\frac{8}{25}$

Score	Frequency
8	5
9	2
10	9
11	8
12	1

5 **a** Given event $A = \{3, 5, 6, 8, 10\}$ and event $B = \{5, 7, 8, 9, 11, 12\}$, find:
i $A \cup B$ **ii** $A \cap B$

b Draw a Venn diagram showing this information.

6 Find the sample space for each situation:

a Tossing two coins

b Choosing a colour from the Australian flag.

7 The table shows the results of an experiment when throwing a die.

a Add a column for relative frequencies (as fractions).

b From the table, find the probability of throwing:
i 3 **ii** more than 4
iii 6 **iv** 1 or 2
v less than 4

Face	Frequency
1	17
2	21
3	14
4	20
5	18
6	10

8 The probability that a certain type of seed will germinate is 93%. If 3 of this type of seeds are planted, find the probability that:

a all will germinate **b** just 1 will germinate

c at least 1 will germinate.

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9 A game is played where the differences of the numbers rolled on 2 dice are taken.

a Draw a table showing the sample space (all possibilities).

b Find the probability of rolling a difference of:

i 3 **ii** 0 **iii** 1 or 2

10 Mark buys 5 tickets in a raffle in which 200 are sold altogether.

a What is the probability that he will:

i win the raffle? **ii** not win the raffle?

b If the raffle has 2 prizes, find the probability that Mark will win just 1 prize.

11 In a class of 30 students, 17 study history, 11 study geography and 5 study neither. Find the probability that a student chosen at random studies:

a geography but not history

b both history and geography

c geography, given that the student studies history

d history, given that the student studies geography.

12 'In the casino, when tossing 2 coins, 2 tails came up 10 times in a row. So there is less chance that 2 tails will come up next time.' Is this statement true? Why?

13 A set of 100 cards numbered 1 to 100 is placed in a box and one is drawn at random. Find the probability that the card chosen is:

a odd **b** less than 30 **c** a multiple of 5

d less than 30 or a multiple of 5 **e** odd or less than 30.

14 Jenny has a probability of $\frac{3}{5}$ of winning a game of chess and a probability of $\frac{2}{3}$ of winning a card game. If she plays one of each game, find the probability that she wins:

a both games **b** one game **c** neither game.

15 A bag contains 5 black and 7 white marbles. Two are chosen at random from the bag without replacement. Find the probability of getting a black and a white marble.

16 There are 7 different colours and 8 different sizes of leather jackets in a shop. If Brady selects a jacket at random, find the probability that he will select one the same size and colour as his friend does.

17 Each machine in a factory has a probability of 4.5% of breaking down at any time. If the factory has 3 of these machines, find the probability that:

a all will be broken down

b at least one will be broken down.

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18 A bag contains 4 yellow, 3 red and 6 blue balls. Two are chosen at random.

a Find the probability of choosing:

i 2 yellow balls **ii** a red and a blue ball **iii** 2 blue balls.

b Find the probability that the second ball is:

i yellow, given that the first ball is blue

ii red, given the first ball is yellow.

19 In a group of 12 friends, 8 have seen the movie *Star Wars 20* and 9 have seen the movie *Mission Impossible 9*. Everyone in the group has seen at least one of these movies. If one of the friends is chosen at random, find the probability that this person has seen:

a both movies **b** only *Mission Impossible 9*.

20 A game of chance offers a $\frac{2}{5}$ probability of a win or a $\frac{3}{8}$ probability of a draw.

a If Billal plays one of these games, find the probability that he loses.

b If Sonya plays 2 of these games, find the probability of:

i a win and a draw **ii** a loss and a draw **iii** 2 wins.

21 A card is chosen at random from a set of 10 cards numbered 1 to 10. A second card is chosen from a set of 20 cards numbered 1 to 20. The 2 cards are placed together in order to make a number, for example 715. Find the probability that the combination number these cards make is:

a 911 **b** less than 100 **c** between 300 and 500.

22 A loaded die has a $\frac{2}{3}$ probability of coming up 6. The other numbers have an equal probability of coming up. If the die is rolled, find the probability that it comes up:

a 2 **b** even.

23 Amie buys 3 raffle tickets. If 150 tickets are sold altogether, find the probability that Amie wins:

a 1st prize **b** only 2nd prize

c 1st and 2nd prizes **d** neither prize.

24 A bag contains 6 white, 8 red and 5 blue balls. If 2 balls are selected at random, find the probability of choosing a red and a blue ball:

a with replacement **b** without replacement.

25 A group of 9 friends go to the movies. All buy popcorn or an ice-cream. If 5 buy popcorn and 7 buy ice-creams, find the probability that one friend chosen at random will have:

a popcorn but not ice-cream **b** both popcorn and ice-cream

c popcorn given that the friend has an ice-cream

26 Ed's probability of winning at tennis is $\frac{3}{5}$ and his probability of winning at squash is $\frac{7}{10}$. Find the probability of Ed winning:

a both games **b** neither game **c** one game.

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9. CHALLENGE EXERCISE

1 In a group of 35 students, 25 go to the movies and 15 go to the league game. If all the students like at least one of these activities, and two students are chosen from this group at random, find the probability that:

a both only go to the movies

b one only goes to the league game and the other goes to both the game and movies.

2 A certain soccer team has a probability of 0.5 of winning a match and a probability of 0.2 of a draw. If the team plays 2 matches, find the probability that it will:

a draw both matches **b** win at least 1 match **c** not win either match.

3 A game of poker uses a deck of 52 cards with 4 suits (hearts, diamonds, spades and clubs). Each suit has 13 cards, consisting of an ace, cards numbered from 2 to 10, a jack, queen and king. If a person is dealt 5 cards, find the probability of getting four aces.

4 If a card is drawn out at random from a set of playing cards find the probability that it will be:

a an ace or a heart

b a diamond or an odd number not including aces

c a jack or a spade.

5 Bill does not select the numbers 1, 2, 3, 4, 5 and 6 for Lotto because he says this combination would never win. Is he correct?

6 Out of a class of 30 students, 19 play a musical instrument and 7 play both a musical instrument and a sport. Two students play neither.

a One student is selected from the class at random. Find the probability that this person plays a sport but not a musical instrument.

b Two people are selected at random from the class. Find the probability that both these people only play a sport.

7 A game involves tossing 2 coins and rolling 2 dice. The scoring is shown in the table.

Result	Score (points)
2 heads and double 6	5
2 heads and double (not 6)	3
2 tails and double 6	4
2 tails and double (not 6)	2

a Find the probability of getting 2 heads and a double 6.

b Find the probability of getting 2 tails and a double that is not 6.

c What is the probability that Andre will score 13 in three moves?

8 Silvana has a 3.8% probability of passing on a defective gene to a daughter and a 0.6% probability of passing a defective gene on to a son. The probability of Silvana having a son is 52%. Find the probability that she has a son, given that Silvana passes on a defective gene.

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